On the Dependency Relationship Between Bids

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Abstract**— Market based contracting introduces increased competition in the power industry, and creates a need for optimized bids and bidding strategies. To maximize the Expected Monetary Value (EMV) of a bid, generation companies (GENCOs) must use good models. Such models should account for factors such as buyers, market mechanisms, and other companies using mathematical models. This paper explores probability density functions for describing a competitor's bids. This probabilistic information is used to formulate a basic competitive bidding profit maximization problem that results in a simple yet informed bidding strategy. Under the defined circumstances, we conclude that the dependency relationship between the density functions describing a competitor's two bids may have no impact whatever on the optimal bid of a company trying to under-bid that competitor.**

Index Terms**—Bidding strategy, decision tree, Expected Monetary Value, dependency relationship.**

I. Nomenclature

The following notations are used throughout this paper.

F_{ij} Operating cost for generation company *i* for generating unit *j*.

S_{ij} Monetary value of operating cost for generation company *i* of generating unit *j*, in \$/MWh.

X_D Total demand in MWh for a given one-hour time period.

X_{ij} Generation capacity in MWh for generation company *i* and generating unit *j* in MWh.

II. Introduction

THIS paper addresses a bidding problem faced by a generation company (GENCO) in a deregulated electric market. Deregulation exposes GENCOs to risks and uncertainties. Electric energy sales by a GENCO depend not only on demand and technical constraints but also on the strategies followed by its competitors. This creates a need for effective decision-support mechanisms that model competitors. In real situations, intelligence about competitors is often uncertain and incomplete, so it is important to develop a bidding model that can flexibly handle various kinds of partial information about competitors' bids. Partial information includes but is not restricted to the dependency relationships among various relevant random variables, such as the bids put forth by a competitor.

The analysis presented here is based on the following question: *What is the optimal bid to make when information about the competitor's bids is uncertain?*

The framework for the analysis is a simplified day-ahead auction where the market is cleared one day in advance on an hourly basis [7]. Producers, GENCOs in this case, submit hourly bids consisting of blocks of energy and their corresponding prices. It is further assumed that this is a single-round auction structure where the market participants only submit the bids once. The price of bid accepted by the buyer is the price it will pay to the winning GENCO to deliver the corresponding block of electric energy.

The next section begins with a simple example model in which perfect information is available. With perfect information, the decision tree approach is applied to the model and yields a straightforward solution. This model is then extended to incorporate probability distributions to express uncertainty in the competitor's bids. We derive an optimal solution to this new model, and discuss the significance of the dependency relationship between bids made by the competitor, GENCO 2.

III. Model 1: Perfect Information

This model consists of GENCO 1 and a competitor, GENCO 2. Both GENCOs are competing to sell X_D megawatt-hours (MWh) of electric energy. GENCO 1 is to determine a bid for an amount and a price that will optimize its expected profit. In a competitive environment, GENCO 1's decision should depend in part on its competitor, GENCO 2. GENCO 1 thus attempts to model GENCO 2 in order to bid optimally against GENCO 2.

The assumptions made in this perfect information model are as follows:

- GENCO 2 has two generators, *A* and *B*. These generators have capacities of X_{2A} and X_{2B} (MWh) respectively, and GENCO 2 would need to use both generators to meet the full demand. Generator *A* is assumed to have a fixed operating cost of S_{2A} \$/MWh in this basic model whereas generator B has a higher cost of S_{2B} \$/MWh, as illustrated in Fig. 1.
- Total demand is X_D for the time period in question.
- GENCO 2 bids at its operating cost. (We may assume that a fixed profit margin is added so that GENCO 2 can make a profit, if desired.) S_{2A} is the

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bid price for the amount of X_{2A} and S_{2B} for the remaining energy needed, which is (X_D - X_{2B}).

- GENCO 1 can meet the demand with one generator, for which the cost of generation is represented by S_{1A}.

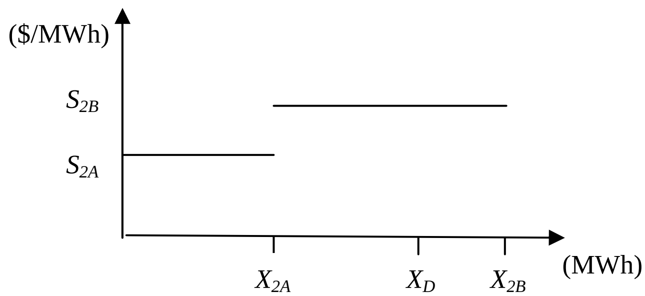


Fig. 1. Price per megawatt-hour as a function of MWh. (An extension to this model would have the function be strictly positively monotonic.)

Both GENCOs submit bids to the Independent System Operator (ISO). The ISO determines the acceptance rules for submitted bids. Bids are accepted starting from the one with the lowest price per MWh, and proceeding to successively higher priced bids until the total need of X_D has been purchased.

Using a profit-maximizing strategy, GENCO 1, with only one generator formulates its maximum profit as:

$$\text{MAX} [\ X_D*(S_{2A}-S_{1A}),\ \ (X_D-X_{2A})*(S_{2B}-S_{1A}),\ \ 0\ ]$$

This function describes the three possible maximum profits for GENCO 1 based on its cost per MWh, S_{1A}. The first expression applies when $S_{1A}<S_{2A}$. In this case, GENCO 1's cost is below the cost of its competitor GENCO 2's generator *A*, which is the generator that produces electric energy most cheaply. (Therefore, it is also below GENCO 2's generator *B*.) Thus, GENCO 1 can bid to sell the total demand, X_D. GENCO 1 can undercut GENCO 2's bid by bidding just below S_{2A}.

On the other hand, if $S_{2A}<S_{1A}<S_{2B}$, it is best to bid just below S_{2B} for the amount of (X_D-X_{2A}), as shown in the 2nd expression of the profit-maximizing function shown earlier. Since GENCO 1's cost in this case is higher than the cost of GENCO 2's generator *A*, the cheaper of the two generators of GENCO 2, it is impractical to undercut that generator by bidding for the total demand because this would incur loss to GENCO 1. Instead of attempting to meet the entire demand, therefore, GENCO 1 should bid for the portion of the demand that the cheaper generator, generator *A* cannot meet at a price that is just low enough to undercut generator *B*. Referring to the 2nd expression, the bid price should therefore be S_{2B}- ε , where ε is assumed to be an insignificant amount whose purpose is to barely undercut GENCO 2's bid of S_{2B}.

When S_{1A} is higher than the cost of GENCO 2's generator *B*, the one that is more expensive to run, then GENCO 1 cannot undercut either of GENCO 2's bids without incurring a loss, so it is better off not bidding at all. This yields the last expression in the profit function, 0.

Based on these facts, how much should GENCO 1 bid? The widely known decision tree approach is presented in the following section as a way to determine the optimal bid for GENCO 1 to submit. This approach will be expanded later when uncertainty is added to the model.

A. Decision Tree Approach

By adopting the decision tree of Figure 2, GENCO 1 calculates the EMV for each leaf to find the most desirable bid, namely the one with the highest EMV.

With reference to the previously defined profit-maximizing function, there are three branches in the decision tree. The first branch denotes GENCO 1's bid of S_{2A}- ε . In order to perform the EMV calculation, GENCO 1 needs to know the probability that its bid will undercut GENCO 2's bid. Since this model assumes complete information, each branch has a certain outcome so that the probability at every leaf is 1. The resulting EMV calculation then for the top branch is simply X_D*(S_{2A}- ε -S_{1A}), where ε represents the amount that GENCO 1 is willing to give up in order to undercut GENCO 2's bid.

The second branch represents the decision to bid at S_{2B}- ε, which would yield a profit rate of S_{2B}- ε -S_{1A}. Since GENCO 1 needs to undercut GENCO 2's more expensive generator in order to win the bid, the amount to bid is X_D-X_{2A}. This yields the EMV expression (X_D-X_{2A})*(S_{2B}- ε -S_{1A}).

The final branch represents the decision that GENCO 1 does not bid, because its cost exceeds the cost of both of the competitor's two generators.

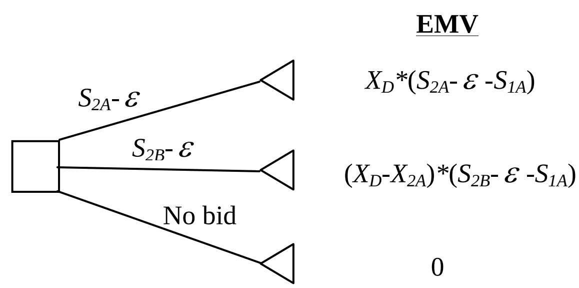


Fig. 2. Decision tree for GENCO 1, given complete information. Mid-range bids make little sense and are not considered.

B. Example

Here is a specific numerical example based on the problem described above. Let the variables be represented by the following values:

X_D = 600 MWh
X_{2A} = 300MWh
$X_{2B} \geq$ 300MWh
S_{2A} = $100/MWh
S_{2B} = $150/MWh
S_{1A} = $40/MWh
ε =1

Since all the variables have exact values, in view of the discussion earlier, the only decision to make is whether to bid at $99/MWh for X_D = 600 MWh, or $149/MWh for X_D-X_{2A} = 300 MWh. It would not make sense to consider any bids lower than $99/MWh for the low bid because GENCO 1 would just make less profit. Similarly, if GENCO 1 decides to try to undercut S_{2B} but not S_{2A}, then GENCO 1 should bid at

\$149/MWh. A bid of \$148/MWh, for example, would make no sense because if a bid of \$148/MWh would be accepted, a bid of \$149/MWh would also be accepted and would lead to higher profit. The decision tree is depicted in Fig. 3. The resulting three EMV calculations indicate that GENCO 1 should bid at \$99/MWh for a total generation of 600MWh to maximize its profit.

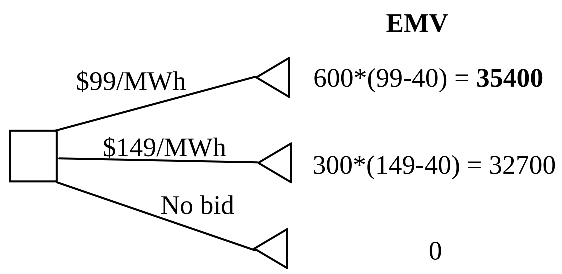


Fig. 3. An example of the decision tree with numerical calculations.

IV. Model 2: Incorporating Uncertainty

In the real world, it is often impossible to obtain various desired data, let alone the competitor's exact bid cost. Most market participants would either use a forecasting tool to estimate the competitor's cost, seek expert knowledge, or study the competitor's strategy and market movements based on historical data. The goal would be to acquire as much reliable, accurate and useful information as possible to include in a decision model.

Suppose that GENCO 1 does not know the precise operating costs of GENCO 2 generators, but has determined probability distributions for the two generators' costs, F_{2A} and F_{2B}. As an example, suppose GENCO 1 models those with the following cumulative distribution functions (see Fig. 5):

F_{2A}: cumulative form of a uniform density function from \$95-105/MWh, and

F_{2B}: cumulative form of a normal density function (tail-trimmed) from \$145-155/MWh.

The profit maximizing function for GENCO 1 remains similar but is represented by distributions instead of exact values. Let us assume for now that the two distributions are independent of each other. Since there is now uncertainty in the bid prices of GENCO 2, the decision tree has to incorporate this uncertainty.

Generally, the decision tree maintains a similar structure to that of Figs. 2 and 3 except that the \$99/MWh and \$149/MWh branches are replaced by sets of branches representing similar, plausible bids. The low bids range from \$94-\$104/MWh and the high bids range from \$144-\$154/MWh. For each plausible high bid b_h, the probability that the higher of GENCO 2's bids, represented by a sample drawn from F_{2B}, is greater than b_h is of importance for EMV computations. Similarly, for each low bid b_l, GENCO 1 must use the probability that its competitor GENCO 2's lower bid is greater than b_l in order to calculate an EMV for bid b_l. A decision tree that includes this information is shown in Fig. 4.

With the decision tree approach, GENCO 1 uses the cumulative distribution functions F_{2A} and F_{2B} for EMV calculations. The decision tree selects a set of sample points from the relevant cumulative density function. Each point becomes a branch of the decision tree. Nine points are chosen from each cumulative density function for this analysis, leading to a tree with 18 branches (Fig. 4).

The formula used for calculating the EMV of bids b_l in the low range is as follows:

$$EMV = 300*(b_l - \varepsilon - S_{1A}) + 300*(b_l - \varepsilon - S_{1A})*P(F_{2A} > b_l)$$

which expresses the fact that 300 Mw will undercut the competitor's high bid, and 300 Mw might undercut the competitor's low bid. The nine low range bids' EMV calculations use this formula by replacing b_l with each bidding price from \$94/MWh to \$104/MWh, and multiplying by the probability $P(F_{2A} > b_l)$ of winning the bid.

The decision to bid high is also plausible because $S_{1A} < S_{2B}$ (represented by F_{2A}). Here, GENCO 1 can sell at most 300 MWh because a bid in the high range represents a decision not to try to undercut F_{2A}, implying that the competitor, GENCO 2, will definitely sell the output of its generator A. The formula to calculate EMV for a branch of the tree for a bid b_h in the high range (\$144 to \$154) is thus:

$$EMV = 300*(b_h - S_{1A})*P(F_{2B} > b_h)$$

The EMV calculations are computed using the above formula by replacing b_h in the EMV calculation with different prices in the range of \$144/MWh to \$154/MWh, and using the corresponding probability $P(F_{2B} > b_h)$.

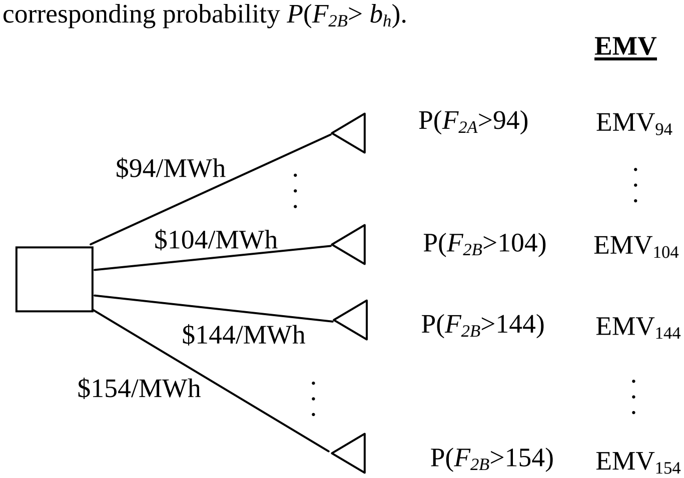
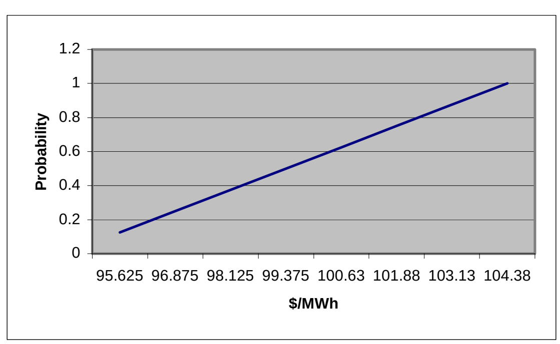


Fig. 4. Decision tree for GENCO 1 with imperfect information about GENCO 2's bids.



(a)

Fig. 5. (a) Cumulative distribution function for F_{2A} (uniform density function). Continued on next page.

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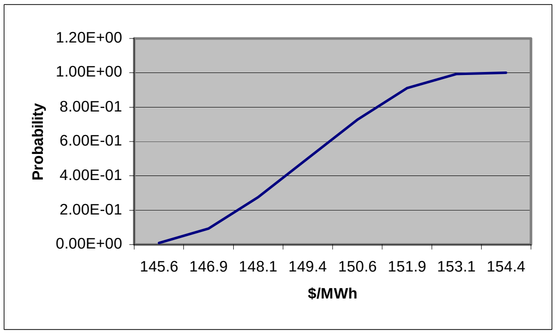

(b)

Fig. 5 continued from previous page. (b) Cumulative distribution function for F_{2B} (tail-trimmed normal density function).

Since the cheaper generator of GENCO 2, generator *A*, has limited capacity, it is evident that GENCO 1 is assured the sale of at least 300MWh if it bids in the low range because generator *A* produces a maximum of 300MWh, leaving another 300Mwh of the 600Mwh demand unfilled. But the chance of selling the full 600 MWh depends on the probability that its bid will undercut the cost of the cheaper generator, generator *A*. For example, if GENCO 1 bids at $97/MWh, it has to consider the probability that the cost of generator *A* will lead GENCO 2 to bid higher than $97/MWh.

Suppose S_{1A} is $40/MWh. The decision tree (Fig. 4) implies that the highest EMV will be $32400 for a bid of $94/MWh for 600MWh. On the other hand, suppose S_{1A} is $90/MWh, then the decision is to bid at $146.25/MWh for 300MWh with a highest EMV of $16689.38. As a third example, if S_{1A} is $97/MWh, the optimal bid does not change, with an EMV of $14612.48 for a bid at $146.25/MWh for 300MWh.

V. Model 3: Increase in Demand

In the preceding discussion with S_{1A} = $40/MWh, the decision was to bid at $94/MWh for the total demand of 600MWh. At first glance it my seem that GENCO 1 might do better by submitting 2 bids, one attempting to undercut generator A and the other attempting to undercut generator B. In fact, a 1-bid strategy is better, as the following shows.

A. 1 bid vs. 2 bids

Suppose GENCO 1, attempting to undercut both generators *A* and *B* of GENCO 2, submits two different bids using, for illustration, the perfect information model of Section III. Since GENCO 2 also submits two bids, GENCO 1's bid that was supposed to undercut generator *B* instead loses to generator *A*. As an example, suppose GENCO 1 submits bids of $99/MWh for 300MWh and $149/MWh for another 300MWh. Then its $99 bid will be accepted, while GENCO 2's $100 bid will also be accepted, and GENCO 1 thus is chosen to meet only part of the demand. If GENCO 1 had made a single bid for the full demand at $99/MWh, it would have won. Thus, GENCO 1 is better off submitting one bid.

However, uncertainty typically arises in the demand for electricity. For instance, extreme weather may cause the demand for electricity to reach a peak. Thus, GENCO 1 has to perform further analysis for the situation where a sharp increase in demand may arise and whether it is still better to submit one bid or two bids.

Referring to the perfect model again, suppose the demand rises to 1000MWh, and GENCO 1 once again considers a strategy of offering $99/MWh for 1000MWh, vs. a strategy of losing 300MWh to generator *2A* while undercutting generator *2B.* Indeed, the EMV calculated for submitting two bids is higher than for submitting one bid at $99 (Fig. 6).

Strategy	Bid ($/MWh)	Bid (MWh)	EMV
1-bid	99	1000	59000
		Total	**59000**
2-bid	99	300	17700
	149	400	43600
		Total	**61300**

Fig. 6. EMV calculations for 1-bid and 2-bid strategies when demand is 1000MWh. Only 400 MWh are offered at $149 in the 2-bid scenario because the competitor will succeed in selling 300 MWh at $100, so the total demand of 1000 MWh will be met.

However, GENCO 1 can do even better by submitting only one bid that undercuts generator *2B*, instead of trying to undercut generator *2A*, as illustrated in Fig. 7.

Strategy	Bid ($/MWh)	Bid (MWh)	EMV
1-bid	99	1000	59000
1-bid	149	700	76300

Fig. 7. EMV calculations for two different 1-bid strategies when demand is 1000MWh. Only 700 MWh are offered at $149 because the competitor will succeed in selling 300 MWh at $100, so the total demand of 1000 MWh will be met.

Thus here again, it is clear that GENCO 1 is better off submitting one bid than 2.

VI. Effects of the Dependency Relationship

Independence between GENCO 2's bids has been assumed in the model that we have discussed so far. The correlation that may exist between the two bids of GENCO 2 is ignored. But in the actual market, correlation and other dependency relationships can exist not only in the fuel prices of generation units using the same fuel type, but also among generation units employing different fuels, as well as for other reasons.

Suppose that generator *A* and generator *B* of GENCO 2 are not independent of each other, but instead have some other dependency relationship. This dependency relationship between the two generators might be describable as positively correlated, negatively correlated, might have a numerical correlation value, etc. Suppose F_{2A} and F_{2B} are positively correlated such that a high sample value S_{2A} drawn from F_{2A} strongly suggests a high sample S_{2B} of F_{2B}, and vice versa.

Referring to the decision tree described earlier (Fig. 4), GENCO 1's decision is not affected by this dependency relationship because each branch whose EMV depends on S_{2A} (the low range bids) does not depend on S_{2B}. Similarly, each branch whose EMV depends on S_{2B} does not depend on S_{2A}. S_{2A} and S_{2B} might be related in some way, but that does not alter GENCO 1's decision because no matter how S_{2B} depends on S_{2A}, the optimal decision made by GENCO 1 is the same.

Similarly, if F_{2A} and F_{2B} are negatively, partially or perfectly correlated, the decision tree remains unchanged with comparable reasoning, thus producing equivalent optimal bids. In conclusion, the underlying dependency relationship between the two random variables describing the two competitor's bids is completely irrelevant in this model.

VII. CONCLUSIONS

The completed analysis has accomplished the objective of the paper by showing that the effect of the dependency relationship between bids can be irrelevant in determining an optimal bid. In the described model, if the domains of probability density function domains for the competitor's two bids are non-overlapping, the optimal bid is not affected by the correlation of the generation units, regardless of their underlying distributions. Where this situation applies, a market participant may ignore a significant form of uncertainty about its competitor without degradation in the quality of its bids.

VIII. FUTURE WORK

As a continuation to this research, our next objective is to extend the results to incorporate a pool of market participants that includes more than two different competing GENCOs. Modeling a group of producers helps to depict a more realistic situation contributing to results that are more applicable in practical decision-making scenarios for power generating companies. In addition to that, the probability density cost functions will be modeled in such a way that the price range may overlap for different generating units. Once again, the dependency relationship among these units will be observed to discover the optimal bidding strategy. A third extended problem that we plan to investigate is the case where the competition has more than 2 generators, and a fourth problem is to extend the decision problem to allow GENCO 1 to make multiple bids at different prices even if it can meet the demand with 1 generator.

In the implemented analysis, the inclusion of uncertainty in terms of the competitor's probability density functions can be classified as 1st-order uncertainty. While one can provide justification for this kind of knowledge about the underlying uncertainty, relaxing this constraint deserves serious consideration. Public or private information based on experts' knowledge, and forecasted and historical data come in various forms, creating the potential for 2nd-order uncertainty, or explicit descriptions of uncertainty about the details of a distribution function on other description of uncertainty. The term indicates the existence of uncertainty about the underlying uncertainty.

We have discussed one type of 2nd-order uncertainty, namely the dependency relationship, which was shown to be safely ignorable. Another type of 2nd-order uncertainty comes into play when distribution functions are not fully specified, leading to envelopes [2] or probability boxes [4] (see Fig. 8). This research will be extended to include this particular type of higher order uncertainty to provide new insights.

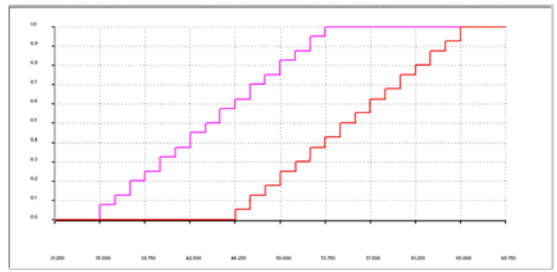

Fig. 8. An example of envelopes or probability boxes distribution function.

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X. BIOGRAPHIES

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