Economic Dispatch: Applying the Interval-Based Distribution Envelope Algorithm to an Electric Power Problem

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Summary

A common way to model uncertainty in the value of a quantity is to use a probability density function (PDF) or its integral, a probability distribution function (CDF). When two such values are combined to form a new value equal to their sum, product, max, etc., the new value is termed a *derived distribution* (Springer 1979). It is well-known that derived distributions may be obtained by numerical convolution, Monte Carlo simulation, and analytically for specific classes of input distributions, under the assumption that the input distributions are independent. It is also possible to obtain derived distributions for specified dependency relationships other than independence. However, it is not always the case that the dependency relationship is known. Thus there is a need for obtaining solutions that are validated with respect to uncertainty about the dependency relationship.

Numerical approaches have the advantage of applicability to a very wide class of distributions. There are two numerical algorithms that have been implemented in software for obtaining solutions to combining distributions that are validated with respect to uncertainty about their dependency. Both also validate their solutions with respect to discretization of the input distributions by using intervals to account for the inexactness of the discretization, eventually producing results that incorporate that inexactness into the separation of the envelopes. One algorithm is Probabilistic Arithmetic (Williamson and Downs 1990), which is implemented in the commercially available software tool RiskCalc (Ferson et al. 1998). The second algorithm is Distribution Envelope analysis (DEnv) (Berleant and Goodman-Strauss 1998). The DEnv algorithm is implemented in a tool, Statool, that extends our previous tool (Berleant and Cheng 1998) by eliminating the need to assume independence. While the Statool and RiskCalc tools have fundamental similarities (Regan et al., submitted) a difference that is relevant to the present problem is that the DEnv algorithm supports, and Statool implements, excess width removal in the underlying interval calculations for expressions in which the true bounds of the expression occur at corners of the rectangle defined by the input intervals. This simple approach frequently works, as for example in the present application. More sophisticated approaches to excess width removal, if implemented, could be incorporated into the software without difficulty since the details of the interval calculations are decoupled from other parts of the software. The result of handling excess width is inferred envelopes that are closer together than they would be if excess width was not handled (Berleant 1993). In this paper we apply the DEnv algorithm to generalize a solution to the well-known *economic dispatch* problem in electric power generation to the case where the dependency relationship between the fuel costs of two generators is unspecified.

1 The Problem

Interval methods have continued to draw the attention of researchers in the power generation community (e.g. Wang and Alvarado 1992; Shaalan and Broadwater 1993; Shaalan 2000). One electric power problem that, as traditionally formulated, is well understood is the economic dispatch problem. In this problem it is desired to determine how much power should be generated by each of two generators, to meet a given level of demand such that total generation cost is minimized. One of a number of approaches to solving this problem is termed LaGrangian Relaxation (Wood and Wollenberg 1996). We added a new dimension to this problem by incorporating uncertainty into the LaGrangian Relaxation technique for solving the problem, by modeling uncertainty in the cost of fuel to run the generators with probability distributions, postulating in addition that the dependency between the two fuel costs of the two generators is unknown (as might occur if one generator burns oil and the other coal). The uncertainties are then propagated through the algebraic expression derived by the LaGrangian Relaxation technique.

First, we specify the cost equations as

$F_1 = v_1(8P_1 + 0.024P_1^2 + 80)$
$F_2 = v_2(6P_2 + 0.04P_2^2 + 120)$

where P_1 and P_2 are the power outputs of generators 1 and 2 in megawatts; v_1 and v_2 are the fuel costs for generators 1 and 2 in \$ per M Btu; and F_1 and F_2 are the generation costs for given power output levels and fuel cost rates. Therefore generation costs change nonlinearly with power output according to the following equations.

$$\begin{aligned} dF_1/dP_1 &= v_1(8 + 0.048P_1) \\ dF_2/dP_2 &= v_2(6 + 0.08P_2) \end{aligned} \tag{1}$$

Solving the problem requires minimizing an objective function,

$F = F_1 + F_2 = v_1(8P_1 + 0.024P_1^2 + 80) + v_2(6P_2 + 0.04P_2^2 + 120)$,

subject to the constraint $P = P_1 + P_2$ where P is the total customer demand for electric power which for this example we take as 400 megawatts. This gives a constraint function

$$P = P_1 + P_2 = 400. \tag{2}$$

By the method of Lagrangian multipliers from calculus, at an extreme value of this objective function,

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2} = \lambda \tag{3}$$

for some λ. This is derived from the Lagrange function L which relates objective function F and constraint (1) according to $L = F + \lambda \cdot P$, which implies $\frac{\partial L}{\partial P_1} = \frac{dF_1(P_1)}{dP_1} - \lambda = 0$ for generator 1 and similarly for generator 2.

From (1) and (3),

$v_1(8 + 0.048P_1) = \lambda = v_2(6 + 0.08P_2)$
$P_2 = 400 - P_1$

and solving simultaneous equations for P_1 gives

$$\begin{aligned} P_1 &= \frac{38v_2 - 8v_1}{0.08v_2 + 0.048v_1} \\ P_2 &= 400 - P_1 \end{aligned} \tag{4}$$

as the most economical amounts of power to generate from generators 1 and 2 to meet the demand (assuming those amounts are within the capacity of both generators). P_1 and P_2 are easily calculated for real values of v_1 and v_2, but given distribution functions for v_1 and v_2, the problem requires evaluating an expression on random variables v_1 and v_2 involving a sum, difference and quotient. Solving it by dividing a difference of random variables by a sum results in excessively wide envelopes on the CDFs for P_1 and P_2 because the same operands occur in both terms, leading to excess width in the underlying interval calculations. Instead the entire expression must be treated as a single binary operation on v_1 and v_2. Figure 1 shows the results given PDFs describing v_1 and v_2.

2 Discussion and Conclusion

Statool currently has certain limitations. Planned extensions include the following.

1. Asymptotic pdf tails. The process of discretizing a pdf into a histogram does not presently allow for the case where a pdf tail trails off to plus or minus infinity. Yet this implies setting definite bounds, though any specific such bounds might be hard to justify. Indeed unusual and extreme values can occur in the electric power domain, as happened for example in the California power crises recently. The solution is to allow the discretization to include open intervals with an end point at ∞ or $-\infty$. This in turn would require the arithmetic operations to be defined on such intervals. Fortunately this is possible, e.g., $[1, \infty) + [1, 2] = [2, \infty), (-\infty, -1] * [-2, -1] = [1, \infty), [1, 2]/[-1, 1] = (-\infty, \infty)$, etc.

2. Partial dependency. While the system currently can calculate either under the assumption of independence, or with no assumption about dependency, partial information about dependency is often present in real problems. Correlation values are a typical example. An example would be prices of different fuels, for which one would expect a generally positive correlation. We are currently working on this issue.

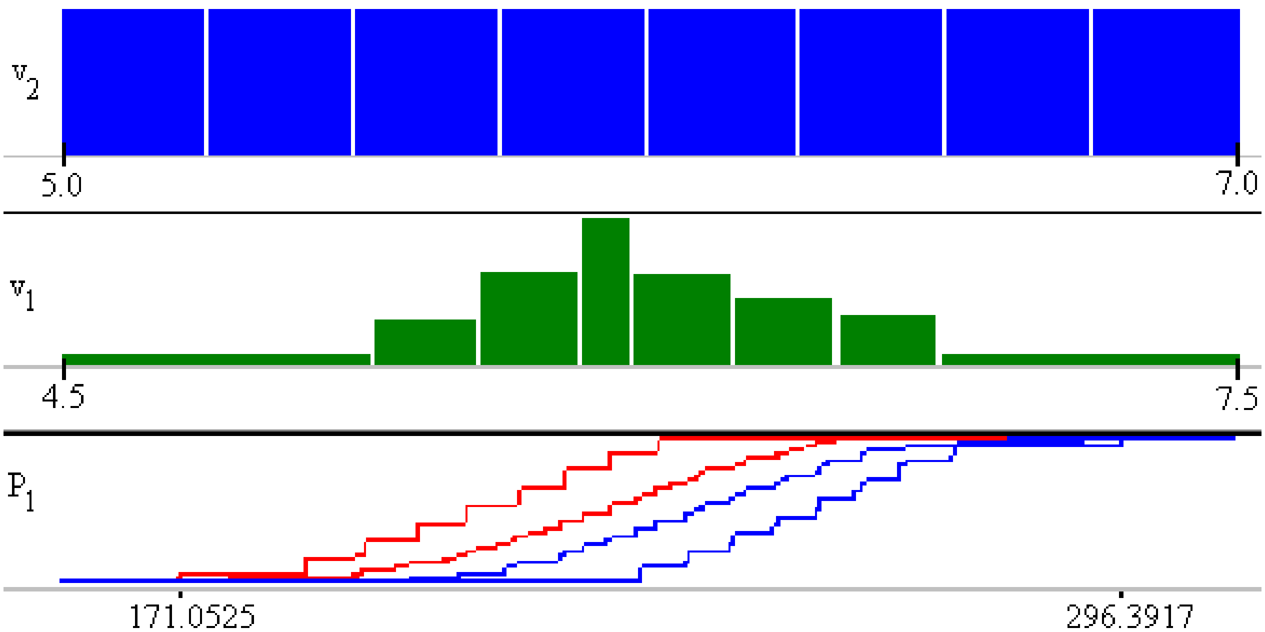

Figure 1. Solution for P_1 of equation (4), given the histogram-discretized PDFs for v_1 and v_2 shown. The CDF for optimum power generation from generator 1 will be within the interior envelopes if the inputs v_1 and v_2 are independent, and within the exterior envelopes regardless of the dependency relationship between inputs v_1 and v_2. When the dependency relationship is not known, the exterior envelopes might be sufficient for a decision, or might point out the need for additional information gathering to sharpen the input distributions and/or identify their dependency relationship sufficiently to support a decision.

The overhead in time complexity due to use of interval calculations is a relevant consideration. For DEnv, time complexity overhead is attributable mainly to the increased time complexity of computing interval operations in place of what would otherwise be numerical ones. Thus a complex excess width removal algorithm would have a correspondingly great effect on run time. The simple method employed in Statool typically adds approximately 25% to the run time, as tested by doing elementary arithmetic operations with and without the excess width handling algorithm, when the dependency relationship between the operands is considered to be unknown. However when the operands are assumed to be independent, using the excess width algorithm leads to a slowdown by an approximate factor of 10, because a higher proportion of the computations done by the program in this case are interval operations, and therefore slowing them has a correspondingly greater effect. These results (25% and 10x) suggest comparing the speed of computation when the dependency relationship as unknown (that is, when the DEnv algorithm is used), with the speed when the operands are assumed independent. When the operands are each discretized into 16 intervals (a 16x16 problem), simple arithmetic operations on the operands take about 30 sec., or 25% more with excess width handling, on an Intel-based PC running at 500MHz. In comparison, assuming independence allows the same problems to run interactively without noticeable delay when excess width handling is not used, and in about 2.5 seconds when it is used.

In the presentation we will explain the DEnv algorithm, and also include explanations and figures showing how assuming independence results in stronger results, while excess width in the interval evaluation of equation (4) leads to weaker results. We will also remark on the implications of envelopes around distributions to decision-makers.

References

- Berleant, D., Automatically Verified Reasoning with Both Intervals and Probability Density Functions (section 4.2), Interval Computations (1993 No. 2), pp. 48-70.
- Berleant, D. and H. Cheng, A Software Tool for Automatically Verified Operations on Intervals and Probability Distributions, Reliable Computing 4 (1) (1998).
- Berleant, D. and C. Goodman-Strauss, Bounding the Results of Arithmetic Operations on Random Variables of Unknown Dependency using Intervals, Reliable Computing 4 (2) (1998) 147-165.
- Ferson, S., W.T. Root, and R. Kuhn, RAMAS Risk Calc: Risk Assessment with Uncertain Numbers. Applied Biomathematics, Setauket, New York, 1998. See also http://www.ramas.com/riskcalc.htm.
- Regan, H., S. Ferson, and D. Berleant, Equivalence of Five Methods for Bounding Uncertainty, submitted.
- Shaalan, H. and R. Broadwater, Using Interval Mathematics in Cost-Benefit Analysis of Distribution Automation, Electric Power Systems Research Journal, Vol. 27, No. 2, pp. 145-152, 1993.
- Shaalan, H., Modelling Uncertainty in Electric Utility Economics Using Interval Mathematics, in M.H. Hamza, ed., *Power and Energy Systems,* Acta Press, 2000.
- Springer, M.D., *The Algebra of Random Variables*, Wiley, 1979.
- Wang, Z. and F.L. Alvarado, Interval Arithmetic in Power Flow Analysis, IEEE Trans. and Power Systems, vol. 7, no. 3, Aug. 1992, pp. 1341-1349.
- Williamson, R. and T. Downs, Probabilistic Arithmetic I: Numerical Methods for Calculating Convolutions and Dependency Bounds, International Journal of Approximate Reasoning 4 (89-158) (1990).
- Wood, A.J. and B.F. Wollenberg, *Power Generation, Operation, and Control, 2nd ed.,* Wiley, 1996.