Electric Company Portfolio Optimization Under Interval Stochastic Dominance Constraints

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Abstract

This paper addresses the problem of market risk management for a company in the electricity industry. When dealing with corporate volumetric exposure, there is a need for a methodology that helps to manage the aggregate risks in energy markets. The originality of the approach presented lies in the use of intervals to formulate a specific portfolio optimization problem under stochastic dominance constraints.

Keywords. Portfolio Optimization, Risk Analysis, Stochastic Dominance.

1 Introduction

All firms face a variety of risks. Each type of risk can affect the firm's financial performance and indeed its valuation. For some purposes, it may be useful to identify and treat these risk units individually. A narrow focus on individual units provides a starting point for estimating an aggregate loss distribution or at least summary risk measures. However, it is the aggregate impact of these risks units on the earnings and value of the firm that is of importance. Because the properties of a portfolio are somewhat different from the sum of its parts, risk management strategies should be derived with reference to those aggregate effects.

This paper looks at relevant corporate risk management strategies with robust methodologies. By robust is meant that any uncertainty consistent with the data or the inputs should be part of the problem statement. One methodology that has been already investigated and implemented by the authors in this framework is interval computing [2], [5]. In this paper, the intent is to move on to the portfolio selection problem, by raising it. In [9], the typical mean-variance portfolio problem is addressed with the interval approach. Another theoretical approach to the portfolio selection problem is stochastic dominance [6]. The major advantage of stochastic dominance over mean variance comes from the ability of the former to take into account irregular or asymetric risky prospects. Therefore, we have focused on the portfolio selection problem under stochastic dominance constraints with intervals.

Raising the problem and describing the framework, the paper is organized as follows. In section 2, the portfolio selection problem and the stochastic dominance concept are introduced. Then, some specifics about electric company portfolios are described. In section 3, interval analysis and probabilistic uncertainty are introduced in order to formulate the mathematical problems (see section 4 to be solved. Finally, two problems are distinguished in terms of interval methodology: the optimization problem under a first order stochastic dominance (FSD) constraint, and under a second order stochastic dominance (SSD) constraint.

2 Problem Statement

2.1 The Performance Indicator

Since the work of Markowitz [13], it is generally agreed that portfolio performance should be measured in two dimensions: profit and risk. In the mean-variance approach, the portfolio selection consists in extracting from all feasible portfolios the efficient ones, namely the portfolios that minimize the risk for each given profit or, equivalently, those that maximize the profit for each risk level. The term "efficiency" refers to Pareto optimality.

Value-at-Risk (VaR) has become a popular measure of market risk, so many discussions of optimization problems involving VaR (instead of variance) appear in the literature. Recall that by definition, with respect to a specified probability level β, the β-VaR of a portfolio is the lowest amount α such that, with probability β, the loss will not exceed α. As a consequence, a β-VaR constraint injected in the portfolio selection problem transforms the initial mean-variance convex problem into a non-convex one. Unfortunately, this means the problem is no longer as tractable. As an alternative measure of risk, conditional Value-at-Risk (CVaR) has been introduced [15]: the β-CVaR is the conditional expectation of losses above the amount α. Now, a β-CVaR constraint injected in the portfolio selection problem remains convex, since the risk is an expectation constraint. Although CVaR has not yet become a standard in the finance industry, it is gaining in the energy industry.

Two other variations on VaR have been developed in recent years: cash flow at risk (CFaR) [8], and earnings at risk (EaR) [7]. EaR, like CFaR, focuses on a specific time period and measures changes in earnings. In the financial industry, EaR is often viewed for a full fiscal year to obtain a comprehensive view including cash flow relating to changes in foreign exchange rates, commodity prices, debts, and investments. CFaR and EaR provide an approach to handling assets which cannot easily be marked-to-market due to accounting rules and which cannot easily be sold, therefore requiring a long time period for risk analysis. Extreme EaR (EEaR) and extreme CFaR (ECFaR) are analogous to CVaR applied, respectively, to EaR and CFaR. For the sake of generality and simplicity, no further distinction is made among all these variations on VaR until section 4.

2.2 Stochastic Dominance Constraints

As a profit/risk performance indicator, stochastic dominance describes a decision procedure that is applicable to risk averters and does not require specification of the individual utility function. The interested reader might consult the literature review provided by Bawa [1] and more recently by Dentcheva and Ruszczynski [6]. Using the robustness made possible by the stochastic dominance methodology, it is expected that pragmatic descriptions of the problem can lead to practical portfolio management conclusions.

By definition, a return of a portfolio X stochastically dominates another one Y to the first order, denoted $X \succeq_1 Y$ if:

$$P(X \leq \beta) \leq P(Y \leq \beta) \quad \forall \beta \in I\!R \tag{1}$$

where $P(X \leq \beta)$ is the cumulative distribution function of X. Thus, if the cumulative distribution of X is equal to or below that for Y for every level of wealth, then prospect X is preferred to (dominates) prospect Y.

The second-order stochastic dominance, denoted $X \succeq_2 Y$, defines a weaker relationship dealing with the *areas below* the cumulative distributions:

$$\int_{-\infty}^{\beta} P(X \leq \eta) d\eta \leq \int_{-\infty}^{\beta} P(Y \leq \eta) d\eta \quad \forall \beta \in I\!R \tag{2}$$

First and second order stochastic dominance have significant similarities in terms of risk management. The first order constraint is equivalent to a continuum of VaR constraints. The second order constraint is equivalent to a continuum of CVaR constraints. Figure 1 illustrates one aspect of the relationship between different orders of stochastic dominance. FSD and SSD are the most commonly used varieties of stochastic dominance. The third most commonly used is infinite-order stochastic dominance. In the context of the present portfolio problem, however, it may be too weak a constraint to recommend portfolios that are guaranteed to be desirable.

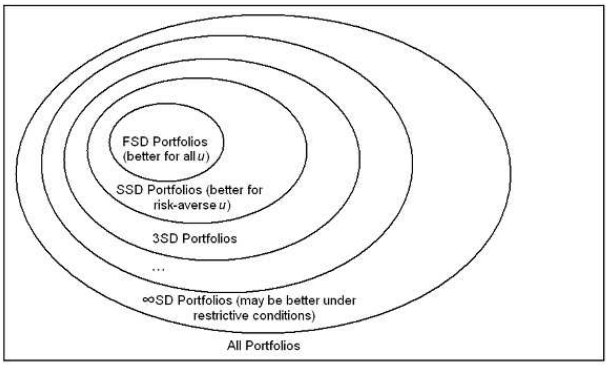

Figure 1: Solution space according the order of stochastic dominance constraint.

2.3 Electric Companies

The problem we address is optimizing the portfolio of an electric company that is exposed to several sources of risks. In a financial context, the two major sources of risk are the market and the volumetric risks. The market risk is related to the impact of price fluctuations on a given portfolio of assets. Volumetric uncertainty mainly comes from hydraulic inflows and from retail demand (in relation to the temperature). In addition, there exists a volumetric risk linked to price fluctuations: with derivatives products, quantities are exercised depending on the price of the initial product, the underlying.

The properties of many electricity markets include poor liquidity and the practical difficulty of storage of electricity. Those two points motivate electric companies to physically balance their portfolio: the difference between demand from the retail market and generation must be accounted for by buying or selling electricity on the pool market for each maturity. Therefore, a volumetric equilibrium constraint will be considered in the portfolio optimization problem:

$$\sum_s x(s)\tilde{D}(s,T,\Theta) + \Delta(T,\Theta) - \sum_g V_g(T,\Theta) = 0 \quad (3)$$

where $\tilde{D}(s,T,\Theta)$ is the random demand of the segment s over the aggregated time period $(T,\Theta) = [T, T+\Theta]$. A typical example of the length of time period Θ would be a month. An example of segment s would be the demand due to small industries in some country, where $x(s)$ is the proportion of this demand in the company portfolio. Then $V_g(T,\Theta)$ represents the generation planning for the aggregated generation units over the aggregated time period (T,Θ). Finally, Δ is a Δ-hedging forward position with delivery at maturity T and covering a period of length Θ. Δ-hedging aims at reducing the sensitivity of the portfolio return to the forward price fluctuation.

The typical integrated portfolio to optimize is composed of three types of business activity: trading, supply and generating. Each of these three portfolio components is itself to be modeled in terms of its own components in order to increase the leverage of the risk managers, as follows.

- Supply optimization uses a maximization function which determines the right proportions of different retail market segments with respect to the retail market itself, given the proportion of the overall portfolio that is allocated to the supply business activity.

- Generation is optimized with a profit maximization function which minimizes the generation costs with respect to a volumetric equilibrium constraint.

- The trading component uses a profit maximization function which helps to hedge when necessary to ensure that the risk limits will be respected, as well as to optimize profit.

The motivation for such an integrated formulation of the problem is that it will increase the feasible set within which the portfolio can be optimized. This in turn allows better solutions to be identified from the resulting larger set of candidate solutions.

3 The Interval Approach

Let us consider a stochastic dominance portfolio optimization model with both an objective function and constraints given by intervals. As a second component of the model, the probabilities of different values x from the interval set $[x]$ of possible values will be used to compute the interval distribution of X. Hence, the problem is to combine global optimization techniques ([11], [10]) with interval probabilities ([3]).

To concentrate our attention to the methodology, rather than the specifics of an electricity company portfolio, it is suggested to start with the following financial portfolio problem:

$$\begin{array}{rl} \max_x & \sum_s x(s)\mathbb{E}(\tilde{R}(s)) \\ s.t. & \sum_s x(s)\tilde{R}(s) \succ_{1,2} \tilde{Y} \\ & \sum_s x(s) = 1 \end{array} \quad (4)$$

where s is a segment of the portfolio, $\tilde{R}(s)$ the given return random vector for s, and $x(s)$ the weight of the segment in the portfolio. $\tilde{Y}$ is a given random variable that represents the risk limit. The stochastic dominance constraint may be of the first or second order.

3.1 Duality

For a given weight $x(s)$ for portfolio segment s, the return is a distribution. For interval-valued inputs, (1) an interval distribution for the return may be obtained, denoted $[\tilde{R}(s)]$, or (2) given a specific distribution d, a range of values for $x(s)$ exist, denoted $[x(s)]$, each with its own interval distribution that encloses d. This range of values for $x(s)$ may be an interval, or may be more complex. A third possibility is that the range might be null.

The duality between interval distributions and interval portfolio weights may allow the overall solution that we seek for the portfolio problem to specify particular portfolio segment weights instead of only intervals for them, at least in many cases. The fact that the weights of the different portfolio segments must sum to one helps to define an optimization problem, the solution to which is convex and thus yields to local optimization in the case of SSD, and non-convex thus requiring global optimization in the case of FSD.

Consider a vector x of three weights, one weight for each portfolio segment. Each weight x_i implies an interval distribution for return, so finding the interval distribution for the entire portfolio requires an algebraic combination of the three interval distributions of the three components. Such algebraic combinations of interval distributions can be done as described in [4]. Let this interval distribution for the portfolio, given segment weight vector x, be d_x. If d_x has the relationship to a reference distribution Y shown in Figure 2, then x is a set of admissible solutions. Our task is to find an optimal weight vector $x_{\max}$ from the admissible set.

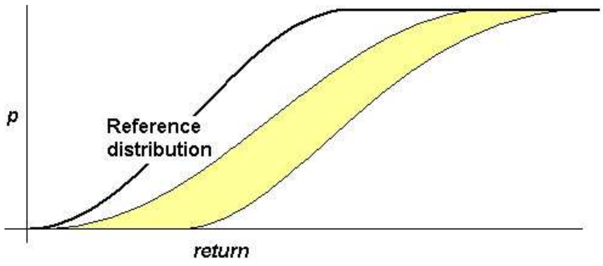

Figure 2: An interval distribution (shaded) that dominates the reference distribution under FSD.

3.2 Weight preferences

As a step toward solving the required optimization problems, it is useful to be able to compare two interval distributions describing the return for a weight vector x. Figure 3 shows a simple case. The interval distribution to the right has a higher return for any given probability level, regardless of what the two true distributions are (since each falls within the bounds of its respective interval distribution). Therefore the weight vector from which it results is a better portfolio choice.

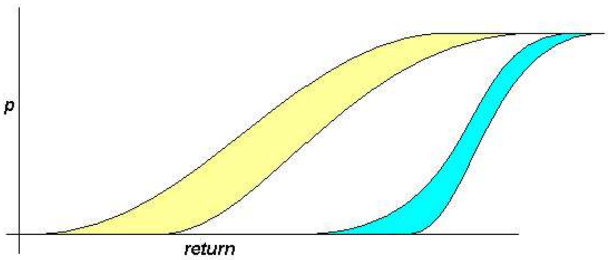

Figure 3: Two interval distributions for return. The one on the right is preferable, so the weight vector corresponding to it is better than the weight vector corresponding to the other.

The situation is not so simple in the case of Figure 4. Here the interval distributions overlap slightly. Thus the two actual unknown distributions, which are constrained to fall within their respective interval distributions, may or may not cross. If they do not cross, FSD holds. If they do cross, the severity of the crossing will be low (in the case shown) so that SSD will hold, but FSD will not. Thus one on the right is preferable under SSD, but might not be under FSD depending on whether or not the actual unknown distributions cross. Therefore the weight vector that resulted in the rightmost interval distribution is better than the weight vector that produced the leftmost interval distribution under an SSD constraint, but might not be under an FSD constraint.

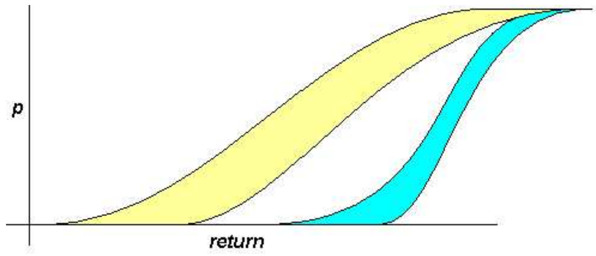

Figure 4: Two interval distribution for return. These overlap slightly, leading to preferability for distributions in the right shaded set under SSD, but not necessarily under SSD.

Another possibility is shown in Figure 5. Here one interval distribution falls completely within the bounds of the other, so it is not possible to determine which dominates the other without more detailed inputs that shrink the interval distributions enough that they do not overlap, if that is possible. Consequently the weight vectors corresponding to the two interval distributions cannot be ranked in terms of their desirability under either FSD or SSD.

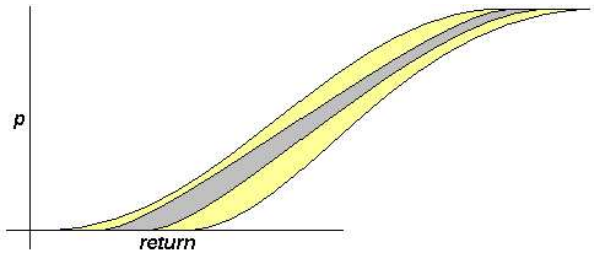

Figure 5: Two interval distribution for return. These are nested, and therefore determining desirability requires further, disambiguating information.

Finally, Figure 6 shows a case where neither return is definitely higher than the other. No amount of further information that might narrow the envelopes of the interval distributions would change that fact. Thus in this case it is not possible to use FSD or SSD to determine a preference for one of them, and consequently it is not possible (under FSD or SSD) to rank the quality of the weight vectors they correspond to.

3.3 Global Optimization

Given an understanding of how two different interval distributions (and hence their corresponding weight vectors) may be compared, it is now possible to consider how to find the best one(s). This is a global optimization problem with a number of interesting

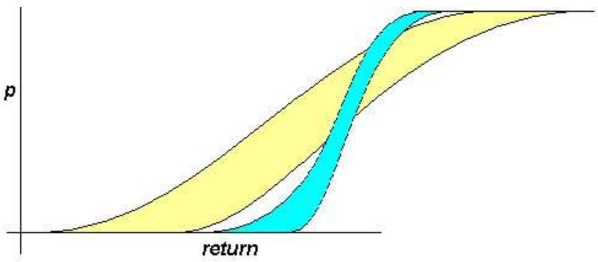

Figure 6: Two interval distribution for return. These cross, so it is not possible to determine which is better (under FSD or SSD constraints) even if perfect input information became available.

aspects that arise from the fact that the distributions are not fully specified, which in turn arises from the lack of complete information about problem inputs. A number of considerations are expected to assist in solving the required optimization problems. These are listed next.

- For convex problems, any weight vector $rx_1+(1-r)x_2$, $0 \leq r \leq 1$, computed from two given weight vectors x_1 and x_2 (that is, any weight which is in the convex set defined by x_1 and x_2), will not be as good as the better of x_1 and x_2. For non-convex problems this may not be true.
- Because the SSD constraint yields a convex optimization problem, one way to easily obtain an optimal solution in some (though not all) FSD problems is to obtain the SSD solution, and simply check if this solution meets the stronger FSD constraint as well. In cases where it does, the SSD solution is the same as the FSD solution. In cases where it does not, global optimization becomes necessary.
- The problem structure requires optimization within an n-dimensional space. Implementations are expected to help shed light on the practical limits to the dimensionality of the problem.

4 Full Formulation

4.1 General formulation

In a strategic framework, but within a market horizon, it is desired to optimize the revenue of a company based on a portfolio of the three business activities of production, supply to the retail market, and trade. The portfolio optimization takes place under a stochastic dominance constraint and a volumetric equilibrium constraint (see Eq. 5). Control variables are the generation planning, the proportion of each retail market within the portfolio (as a result of the optimal bid prices), and the hedging positions. The strategic aspect of the problem comes from the intent to determine the proportion of each retail market. The philosophy is not to reach a very detailed schedule of power plants, but to solve a global optimization problem with several random variables and aggregated portfolio.

$$\begin{array}{l} \max_{(x,\Delta,V)} \mathbb{E}\left(\tilde{CF}(x,\Delta,V)\right) \\ s.t. \left\{ \begin{array}{l} \tilde{CF}(x,\Delta,V) \succeq_{1,2} \tilde{Y} \\ \sum_s x(s)\tilde{D}(s,T,\Theta) + \Delta(T,\Theta) - \sum_g V_g(T,\Theta) \\ \quad = 0 \end{array} \right. \end{array} \tag{5}$$

and:

$$\begin{array}{l} \tilde{CF}(x,\Delta,V) \\ = \sum_{(T,\Theta)} \sum_s x(s)\tilde{R}(s,T,\Theta) \\ \quad +\Delta(T,\Theta)\tilde{S}(T,\Theta) - \sum_g V_g(T,\Theta)C_g(T,\Theta) \end{array} \tag{6}$$

$\tilde{R}(s,T,\Theta)$ equals the random demand times the average price for this particular retail market. $\tilde{S}(T,\Theta)$ is the random average spot price over the time period, and $C_g(T,\Theta)$ the generation cost for the unit g. The above equilibrium constraint is an equality constraint that could be replaced by a bound constraint $[-\epsilon,\epsilon]$ if it facilitates the solution search.

4.2 FSD

In the case where the first order stochastic dominance constraint is to be studied, the mathematical problem to be solved is non-convex. Hence, its resolution requires global optimization techniques: state variables (the weights of each segment) are represented by intervals, and the optimization process consists in narrowing as much as possible the prior intervals. However, a first order stochastic dominance constraint seems easier to use than a second order one since it is possible to bound all probability distribution functions consistent with the input data by two cumulative distribution functions. Such bounds will enable the portfolio return to satisfy a first order stochastic dominance constraint with respect to a given reference cumulative distribution function. In other words, both the state variables and the probabilities can be intervals in this formulation of the problem.

4.3 SSD

In the case where the second order stochastic dominance constraint is to be used, the mathematical problem turns out to be convex. As a consequence, the optimization problem to be solved is not as complex as in the case of first order stochastic dominance. This should strongly decrease the computation time relative to the case of first order stochastic dominance.

Because the SSD constraint yields a convex optimization problem, one way to easily obtain an optimal solution in some (though not all) FSD problems is to obtain the SSD solution, and simply check if this solution meets the stronger FSD constraint as well. In cases where it does, the SSD solution is the same as the FSD solution. In cases where it does not, global optimization becomes necessary.

4.4 Dimensionality

For the simplest abstraction of the problem, the decomposition of the full portfolio is into 3 major segments: generation, supply and trading. For the full portfolio selection problem, involving retail market proportions, delta hedging and aggregated generation scheduling, n is about 100. A modified version of the full problem would reduce the dimensionality of the problem by aggregating for example the generation scheduling into one distribution instead of 6 (aggregated) units, in which case n is now about 50.

- For the simplest abstraction (3 dimensions), the optimization space is likely to have a manageable number of local maxima (and minima) compared to the general problem in which optimization spaces may be of arbitrarily high dimensionality.
- For full-scale problems, global optimization is expected to be a more significant issue. For these problems, several software packages are available for global optimization.

5 Conclusion

The problem of reducing aggregated market risk has been formulated in terms of optimal portfolio selection under stochastic dominance constraints with intervals. This formulation is expected to yield useful results itself, and also lead to additional interesting questions of potentially great practical applicability. One possible direction is infinite order stochastic dominance [12]. This is the weakest stochastic dominance constraint. Hence it admits the largest feasible set of solutions, which for this application is the set of potentially desirable portfolios. It may be valuable to investigate how infinite order stochastic dominance compares with first and second order stochastic dominance for the electric company portfolio problem.

Regardless of the type of stochastic dominance considered, there is the possibility that no one vector of weights will dominate all others. In such cases a decision must nevertheless be made. Decision-making in the context of electric company financial problems may be of interest in these cases. Work on this is of current interest [2] and may become relevant to this project at a later stage.

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