Information Gap Decision Theory as a tool for strategic bidding in competitive electricity markets

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***Abstract*—Market based contracting introduces increased competition in the power industry, and creates a need for optimized bids and bidding strategies. To maximize the Expected Monetary Value (*EMV*) of a bid, generation companies (GENCOs) must strive to use models better than their competitors. Such models should account for factors such as buyers' market power, market mechanisms, other competitors, substitutes, and equipment status. This paper explores bounds on the probability distribution describing the competitors' bids. This weak probabilistic information is used to formulate a basic competitive bidding problem. In this environment, the bidder is expected to perform better provided they are *informed* about factors impacting the competitor's bids. However, the acquisition of this kind of information involves costs that may exceed the expected benefit. Therefore, the bidder must decide whether or not to acquire information to alter the optimal bid. This paper explores use of Information Gap Decision Theory to quantify severe uncertainty. The value of additional information is compared under a more informative info-gap model where it determines the demand value of the information.**

***Index Terms*—Bidding strategy, 2nd-order uncertainty, Expected Monetary Value, Information Gap, value of information.**

I. Nomenclature

The following is the list of notations used throughout this paper. Other notations, especially those needed in describing the information gap model, are given in the relevant sections.

F_{ij} Operating cost for generation company *i* for generating unit *j*.

G_{ij} Generator for generation company *i* of generating unit *j*. (This notation is introduced to refer to the physical unit itself as opposed to the cost, which is F_{ij}).

X_D Total demand in MWh for a given one-hour time period.

X_{ij} Generation capacity of G_{ij} in MWh.

B_j Bid price for *j* number of bids.

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II. Introduction

THIS paper addresses a bidding problem faced by a generation company (GENCO) in a dynamically restructured electricity market. In this environment, GENCOs are exposed to risks and uncertainties. Electric energy sales by a GENCO depend not only on demand and technical constraints but also on the strategies followed by its competitors. This creates a need for effective decision-support mechanisms that model competitors. In real situations, intelligence about competitors is often uncertain and incomplete, so it is important to develop bidding models that can flexibly handle various kinds of partial information about competitors' bids. Partial information includes but is not restricted to the dependency relationships among various relevant random variables, such as the bids put forth by a competitor. In the problem addressed in this paper, two GENCOs are competing to supply a fixed electricity demand. Taking GENCO 1's perspective, the bidding strategy against GENCO 2 is formulated to include some past data together with expert judgment about GENCO 2's bidding behavior. Using this imprecise information, we will attempt to quantify the uncertainty GENCO 1 faces and how to improve the situation by acquiring new information. The acquisition of information will be justified or not by its cost and its contribution to the process of developing a bid.

Most publications that propose methods to estimate the bidding behaviors of rival participants are developed based on probabilistic analyses [11,13]. However, these probabilistic techniques do not handle fuzzy or heuristic information. Research on that has investigated techniques such as fuzzy set based methods [6], possibility theory [14], and intelligent trading agents, such as genetic algorithm, genetic programming, and finite state automata that are utilized for developing adaptive and evolutionary bidding strategies [7,8].

This paper proposes information gap (info-gap) decision theory (IGDT) [1] to develop bidding strategies for generation companies. IGDT is useful when decisions must be made under severe uncertainty. A non-probabilistic quantifier of uncertainty that makes no underlying assumptions about the structure of the uncertainty, an info-gap model aims to concentrate on what *is known* and what *could be known*. Given very sparse information, a "robustness" function will be introduced to describe immunity to failure. This function helps to facilitate the study of various trade-offs inherent in the decision.

The analysis presented here is based on the previous work [3] developed by the authors to address the following question: *How is a company to bid when information about the competitor's bids is highly uncertain?* The framework for the analysis is a simplified day-ahead auction where the market is cleared one day in advance on an hourly basis [9]. Producers, GENCOs in this case, submit hourly bids consisting of blocks of energy and their corresponding prices. It is further assumed that this is a single-round auction structure where the market participants only submit the bids once. The price of a bid accepted by the buyer is the price it will pay to the winning GENCO to deliver the corresponding block of electric energy, which is also known as *discriminatory pricing*.

The following section begins with the problem formulation and explains how the Expected Monetary Value (*EMV*) for each bid is bounded. An info-gap model is then developed based on the resulting *EMV* bounds. Acquisition of additional information can be expensive and depends on the demand value of information. A comparison on how much gain we can get from the extra information will be discussed to justify the worth of acquiring possibly costly information. Finally, we summarize the results discovered as well as the directions for future work.

III. PROBLEM FORMULATION

This problem is formulated with two generation companies, GENCO 1 and a competitor, GENCO 2. Both GENCOs are competing to sell X_D megawatt-hours (MWh) of electric energy. GENCO 1 is to determine a bid for an amount and a price that will serve its profit-making interests. In a competitive environment, GENCO 1's decision should depend in part on its competitor, GENCO 2. GENCO 1 thus attempts to model GENCO 2.

In general, the basic elements of contract bidding include direct labor costs, mark up or return, overhead, and profit. If GENCO 1 intends to undercut GENCO 2 and win the sale, then GENCO 2 generation cost has to be included in the model. Various traditional methods can be used to build this model; one such method is to use random variables with applicable distributions to represent the unknowns. While these approaches may be able to produce results, the dependability of these results on the underlying assumptions about the details of the distributions can make them problematic. To force assumptions in order to make the problem tractable is undesirable.

Given lack of knowledge about the generation costs and the relationships among these variables, a natural approach is uncertainty quantification. Instead of using specific distribution functions to model the cost functions, F_{2A} and F_{2B}, we employ probability boxes to model the incompleteness of the available information. In other words, the uncertainty is described by distribution functions together with error bounds, as shown in Figs. 1-2.

Given the 2nd-order uncertainty for GENCO 2's cost functions, GENCO 1 has the options to submit one bid or two bids. By submitting one bid, GENCO 1 decides whether to underbid G_{2A} or G_{2B}. Two-bid submission involves trying to underbid both generators. These 3 scenarios are simulated and analyzed to determine the optimal one given the total demand of X_D=1000MWh with F_{1A}=\$40/MWh. The generation capacity for each generator is as follows:

$X_{1A} \geq$ 1000MWh
X_{2A} = 300MWh
$X_{2B} \geq$ 700MWh

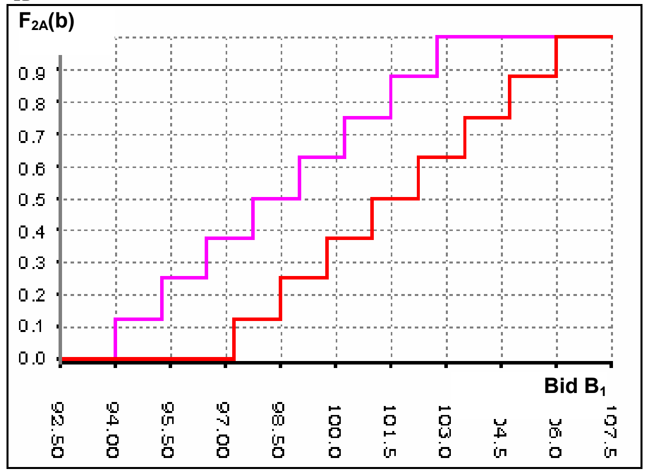

Fig. 1. GENCO 2 cost function F_{2A} for G_{2A}.

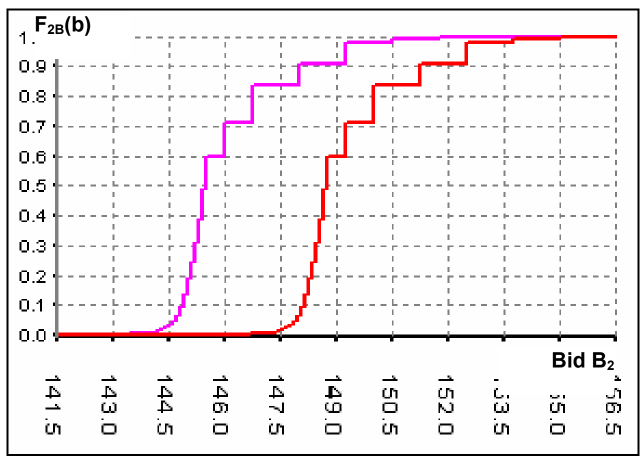

Fig. 2. GENCO 2 cost function F_{2B} for G_{2B}.

A. Scenario 1: Attempt to underbid G_{2A}

In this scenario, GENCO 1 formulates its analysis by ignoring the existence of G_{2B}. With only one goal in mind, that is to outbid G_{2A}, two different decisions are made with respect to the cost of G_{2A}, which is represented by F_{2A}. If $F_{2A} > B_1$, GENCO 1 can sell all 1000MWh of electricity because G_{2A} has higher cost. On the other hand, if $F_{2A} < B_1$, then GENCO 1 definitely loses to G_{2A} and thus, can at best sell 700MWh of electricity. Since F_{2A} is described by a probability box with error bounds (Fig. 1), the *EMV* calculated will be bounded by an interval. An example of how the *EMV* values are calculated is shown in Fig. 3.

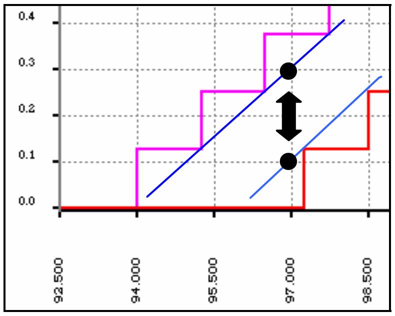

Fig. 3. GENCO 2 cost function, F_{2A}, for G_{2A}.

Assuming that bid B_1=\$97/MWh, the cumulative probability range for F_{2A} definitely less than B_1 is [0, 0.1], with a width and therefore in this case a probability of 0.1. The cumulative probability range for F_{2A} definitely greater than B_1 is [0.3, 1.0], with a width and therefore a probability of 0.7. The cumulative probability range over which it is not known whether F_{2A} is less than or greater than B_1 is [0.1, 0.3], which has width and therefore probability 0.2. The following calculations may be performed, where the low bound of the *EMV* is denoted by *EMV*(*low*) and the high bound is represented by *EMV*(*high*).

1) *Case 1:* $F_{2A} < B_1$ (definitely)

   Profit = 700*(97-40)*(0.1-0) = 3990

2) *Case 2:* $F_{2A} > B_1$ (definitely)

   Profit = 1000*(97-40)*(1.0-0.3) = 39900

Case 3:

The *EMV* of this case can vary from a case 1-like minimum of 700*(97-40)*(0.3-0.1) = 7980 to a case 2-like maximum of 1000*(97-40)*(0.3-0.1) = 11400.

Since either case 1, 2, or 3 will turn out to apply, the *EMV* of a bid of \$97/MWh is the sum of the three *EMV*s of the three cases, or 3990+39900+[7980, 11400]=[51870, 55290].

Doing the same computations on other bid prices, it turns out that the bid with the highest potential *EMV* occurs at \$96.25/MWh with an *EMV* of 55714 and the bid with the highest guaranteed minimum is \$94/MWh with an *EMV* of 54000, as shown in Fig. 4. The upward trend toward the right-hand side of Fig. 4 would be the left-hand side of an analogous figure illustrating scenario 2, described next.

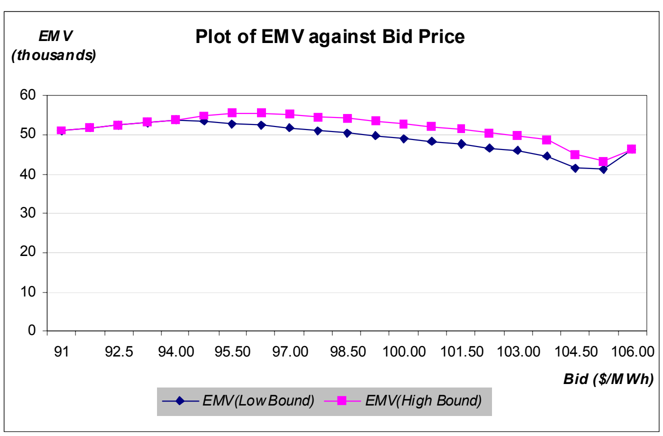

Fig. 4 Plot of *EMV* bounds (in thousands) against bid prices (in \$/MWh).

B. *Scenario 2: Attempt to underbid G_{2B}*

In this scenario GENCO 1 attempts only to underbid GENCO 2's more expensive generator, perhaps thinking that the resultant high rate of return per MWh if that 700 MWh bid is accepted will more than make up for the 300 MWh block that will not be sold because GENCO 2's less expensive generator G_{2A} wins that block.

The *EMV* calculations are performed similarly to the previous scenario except that the cost function for G_{2B} is used in this case instead of the cost function for G_{2A}. It turns out that the highest *EMV* that may be obtained with certainty under this scenario is at a bid of \$143/MWh, for an *EMV* of 72,100. However, a bid of \$143.90/MWh leaves open the possibility that we may enjoy an even higher *EMV* than that because then the *EMV* is within the interval [71664.42, 72195.25] (Fig. 5).

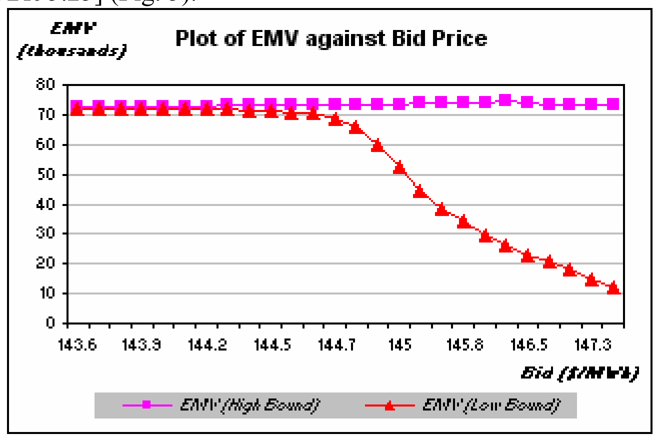

Fig. 5 Plot of *EMV*(*low*) and *EMV*(*high*) based on the cost function of G_{2B}.

C. *Scenario 3: Attempt to underbid both G_{2A} and G_{2B}*

Here, GENCO 1 submits 2 different bids (B_1 and B_2) based on an attempt to underbid the two generators of GENCO 2. If it is definite that $B_1 > F_{2A}$, GENCO 1 does not sell the 300 MWh block because it did not underbid G_{2A}. When on the other hand it is definite that $B_1 < F_{2A}$, GENCO 1 sells the 300 MWh block. When it is not definite whether $B_1 > F_{2A}$ or $B_1 < F_{2A}$, the *EMV* cannot be exactly known because the distribution of Fig. 1 is not exactly known. As for the attempt to underbid GENCO 2's higher cost generator G_{2B}, when it is definite that $B_2 > F_{2B}$, once again GENCO 1 loses (and hence need not even bother to bid). However, when it is definite that $B_2 < F_{2B}$, GENCO 1 can sell 400MWh of electricity at a price of B_2 because it sells 300MWh at price B_1 and loses another 300MWh to G_{2A}. Working through the computations gives a maximum certain *EMV* of 57400, obtained by submitting a bid of \$94/MWh for 300MWh and a bid of \$143/MWh for 400MWh. However, higher bids can result in interval-valued bounds on *EMV*s for which the high bound is higher than 57400 although the low bound would be lower than that.

D. *Summary Based on the 3 Scenarios*

The results clearly show that the best scenario of the three is the second, in which GENCO 1 attempts to underbid only G_{2B}. Thus, this scenario should be used to guide the bid. However this still leaves open the question of exactly what to bid given the uncertainty present. Perhaps additional information will reduce uncertainty and allow us to better determine a value to bid. The next section introduces the info-gap model to quantify the uncertainty described by the envelopes bounding the cost function of G_{2B} (Fig. 2) in order to assist GENCO 1 in determining the best bid value.

IV. The Info-Gap Model

Information Gap Theory [1] is useful for making decisions in cases where uncertainty is present and severe. For example, distributions may be not fully specified, as in Figs. 1-2.

Suppose that we wish to ensure that the *EMV* of a bid (corresponding to the expected profit) meets or exceeds a given minimum value. An information gap model helps to identify bids that meet that requirement. More interestingly, the model also identifies the uncertainty-reducing information that would need to be obtained to ensure that other, possibly more desirable bids, meet that requirement. An example of such a potentially more desirable bid would be one that corresponds to a wide range of possible *EMV* values, some quite high and desirable, and others below a minimum tolerable *EMV*. For example, in Figs. 5-6, the bidder may enjoy a high *EMV* of 74200, at a bid of \$145/MWh, but that bid may also result in an *EMV* of 29680 if the true curve happens to be the lowest *EMV* curve shown. This staggering range can be reduced with more information that reduces the amount of uncertainty in the model. A comparison between the cost of obtaining such information and its benefits should be performed to decide on whether to obtain the information or not.

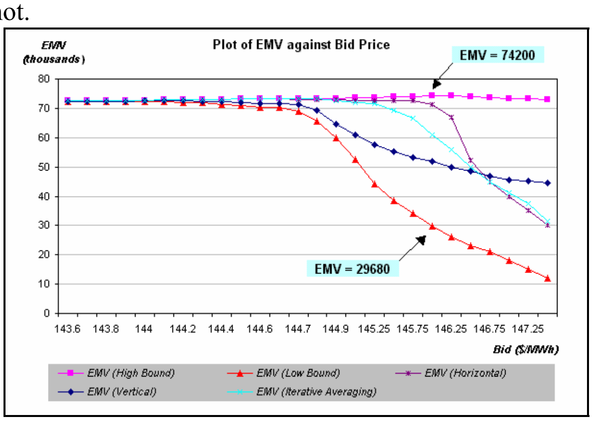

Fig. 6. A wide range of possible *EMV* values for a given bid.

An information gap model for this example problem may be specified as follows.

1. ***Decision variable.*** This is our bid B_2 in *\$/MWh*.
2. ***Uncertain variable.*** Define a CDF for the competitor's bid that serves in the role of nominal best guess. Any CDF judged to fill this role could be used. For purposes of illustration we use "horizontal averaging" of the left and right CDF envelopes of Fig. 2, giving the intermediate curve of Fig. 8. In horizontal averaging, for each vertical axis value y_i, the corresponding horizontal axis values of the left envelope, B_l, and of the right envelope, B_r, are averaged, giving a value $B_i=(B_l+B_r)/2$. The point (B_i, y_i) is on the average CDF curve, which may be plotted as precisely as desired by using an appropriate set of values for *i*. The average CDF serves as a nominal best guess CDF. Our current work suggests that considering other averaging methods as well, but the structure of the following discussion is independent of what averaging method is used. Horizontal averaging as just defined weights the left and right envelopes equally. However the weights of the envelopes could potentially be anything between 0 and 1 (the weights must add up to 1). These weights effect the averaging computation. Accounting for weights generalizes the averaging formula to $B_i=(wB_l+(1\text{-}w)B_r)/2$. Let the uncertain variable in the info-gap model be the weight *w* of the left envelope, with the weight of the right envelope then being 1-*w*. Then Fig. 7 describes the *EMV* values calculated from the CDF envelopes of Figs. 2 & 8.
3. ***Nominal value of uncertain variable.*** There seems to be no particular reason to prefer weighting one envelope more than the other when doing horizontal averaging, so the default nominal value of weight *w* is $\tilde{w}=0.5$.
4. ***Uncertainty parameter.*** The amount of uncertainty in the model, *α*, is the amount of deviation from the nominal value of the uncertain variable that is to be considered. In this model, that is the amount of deviation from $\tilde{w}=0.5$. In the worst case, this might be ±0.5, giving a range of weights from 0 to 1. Further information might shrink the uncertainty parameter to a subset of ±0.5, and it might be necessary to obtain such information to ensure goals are met. Determining this is the goal of the information gap analysis.

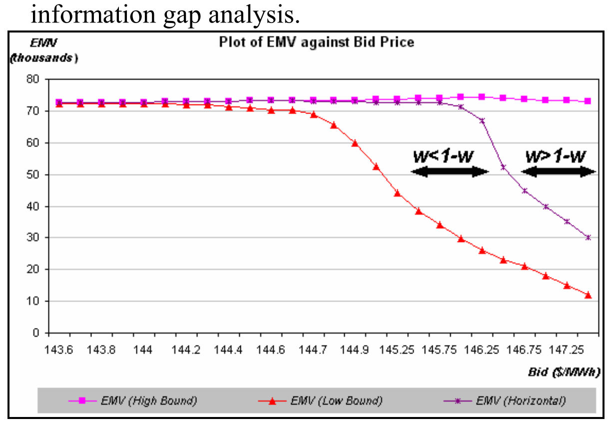

Fig. 7. *EMV* curves corresponding to the left envelope of Figs. 2 & 8 (lowest curve), the right envelope (highest curve), and the horizontal average of the left and right envelopes.

5. ***Uncertainty model.*** This is the function $U(\alpha,\tilde{w})$ that describes the amount of uncertainty in the uncertain variable *w* in terms of its nominal value $\tilde{w}$ and uncertainty parameter *α*. Consistent with points 2-4 above, we have $U(\alpha,\tilde{w})=\{w: w=|\tilde{w}+\alpha|\}$.

6. ***Reward function.*** The reward is the *EMV* of a bid. It is determined by the bid value and the *EMV* curve that applies. In this problem, the *EMV* curve is uncertain, so the worst possible *EMV* curve is used, to allow the reward function to provide the minimum *EMV* that the bid could be associated with, as required by the info-gap analysis. The worst possible *EMV* curve is in turn determined by the leftmost possible CDF curve for the competitor's bid. This curve is found by horizontal averaging with an averaging weight of $\widetilde{w}+\alpha$. Using this curve is consistent with the ultimate goal of designing the bid and any necessary information-seeking activities to ensure at least a minimum *EMV*. Thus reward function R is defined by:

$$R(B_j,w) = EMV\,(B_j, w\cdot\overline{F_{2B}}(B_j) + (1-w)\cdot\underline{F_{2B}}(B_j))$$

Where $\overline{F_{2B}}(B_2)$ is the highest possible envelope around the function F_{2B} (i.e. the left envelope in Fig. 2), and $\underline{F_{2B}}(B_2)$ is the corresponding lowest possible envelope (i.e. the right envelope).

7. ***Critical reward.*** This is the minimum acceptable value of the reward function, call it r_c. The results of an information gap analysis differ depending on the value assigned to r_c, as we will see. This minimum acceptable value is an input to the model.

8. ***Robustness function.*** This function, $\hat{\alpha}(b,r_c)$, returns the greatest value of uncertainty parameter α for which falling below the critical reward r_c is not possible in the model. It therefore measures the ability of the model to deliver acceptable reward in the presence of uncertainty, hence the term *robustness*. Its value is therefore dependent on acceptable reward r_c. It is also dependent on the bid B_2, because the reward is dependent on B_2.

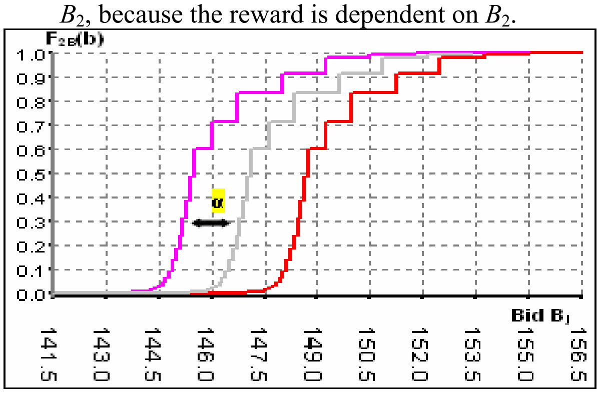

Fig. 8. Best-guess curve between the left and right envelopes computed by horizontal averaging, and the maximum uncertainty α, showing that the space of plausible curves is within the envelopes.

Fig. 9 gives information about $\hat{\alpha}(b,r_c)$ for a range of bid values, and a specific value of r_c, which would be a business decision provided as a model input. For bid values toward the left of Fig. 9 (denoted as region 'X'), the *EMV* is above r_c regardless of which *EMV* curve is considered, so r_c will be safely met for any of those bids.

However it may be desired to consider bidding higher in order to reap the potential opportunity for greater gain due to potentially higher *EMV* values such as, for example, the peak feasible *EMV* of $146.25/MWh for region 'Y'. This is simply an analytical elaboration of the intuition that the higher the bid, the greater the profit if the bid is successful, but the higher the chance that a competitor will undercut the bid resulting in no profit. Thus, bids in the range designated by 'Y' in Fig. 9 are not guaranteed to have an *EMV* of at least r_c unless new information is obtained that rules out values of w that are too close to 0 (thereby moving the worst-case *EMV* curve upward). Note that from Fig. 9, a bid of $146.25/MWh, which would possibly result in an *EMV* as high as 74315.5, is infeasible if we want to be sure to get a reward at least as high as critical reward r_c.

Finally, bids so high that even the top *EMV* curve falls below r_c (somewhere to the right of the domain shown in Fig. 9) are infeasible in that they will definitely result in an *EMV* below r_c.

To summarize, the result of an information gap analysis is a validation (or invalidation) of a bid value based on whether its *EMV* meets minimal requirements. However, the analysis does not tell us what the best bid is. This leads us to the next section.

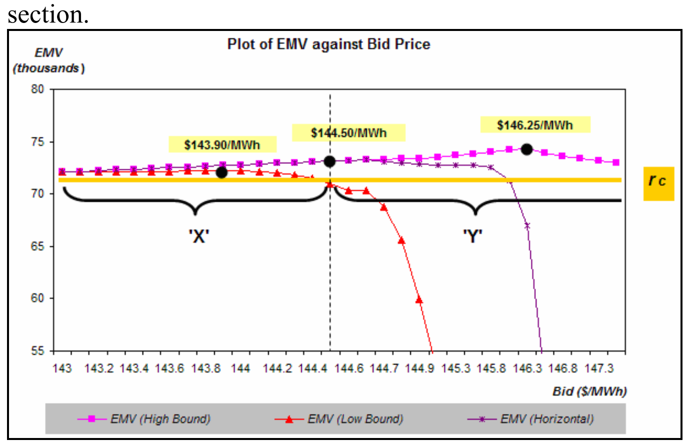

Fig. 9. Critical reward separates the *EMV* curves into regions.

V. Determining the Bid

There are several ways to determine the bid, including maximizing worst-case *EMV*s, maximizing expected *EMV*s, and converting *EMV*s to utilities using risk profiles. Fig. 9 shows that the set of bids that ensure a minimum critical reward of r_c is in region 'X'. If GENCO 1 is risk-seeking, it would bid based on the high *EMV* curve, which predicts a yield of 73150 from a bid of $144.5/MWh. From a risk-averse viewpoint, GENCO 1 would bid $143.9/MWh for an *EMV* value of 72195.25, the maximum of the low *EMV* curve.

Another approach is to seek information that will reduce the uncertainty in the model, thereby enabling a more informed bid. However acquisition of information requires an expenditure of resources. The question then is how to gauge the value of the information. Suppose new information leads us to a more informative info-gap model U_{new} which is a subset of $\mathrm{U}\,(\alpha,\widetilde{w})=\{w\colon w = |\widetilde{w}+\alpha|\}$ mentioned earlier. Since U_{new} is more informative, then this implies a model which is likely to be more robust. In the bidding example, a larger range of bids is likely to be feasible (Fig. 10).

Clearly more information is good to have. This can be quantified. Suppose that the additional information results in improved bounds on the behavior of the system as shown in Fig. 10. For the same critical reward r_c with a pessimistic decision criteria, we can bid at a higher price, with the increase denoted by *ΔB*. On the other hand, if we do not bid higher but instead use the same bid shown earlier in Fig. 9, we enjoy a higher minimum *EMV*. The change in *EMV* is shown in Fig. 10 by *ΔEMV*. From the graph, the exact values are determined.

ΔB = \$144.70/MWh - \$144.40/MWh = \$0.30/MWh

ΔEMV = \$72268.80 - \$71447.93 = \$820.87

Further work is needed to connect this kind of analysis to the amount that GENCO 1 should be willing to pay for information.

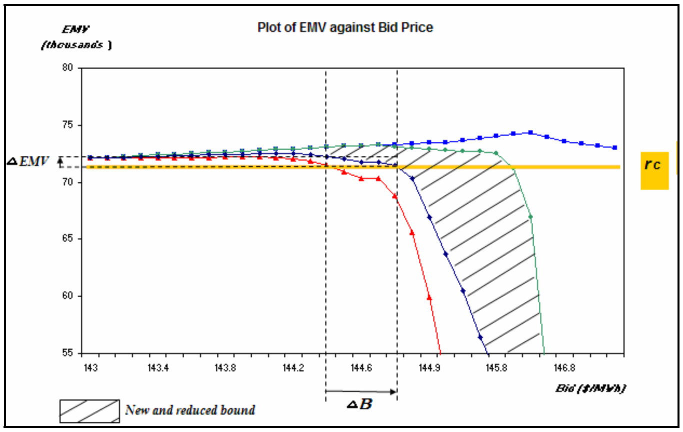

Fig. 10. A plot of the original info-gap reward curve versus the more informative info-gap reward curve that is found with additional information.

VI. CONCLUSIONS

This paper shows how Information Gap Decision Theory (IGDT) can serve as a decision support tool that assists in quantifying severe uncertainty when information is scarce and expensive. It can help decision makers to develop preferences, assess risks and opportunities, and seek information, given a minimum required level of reward. This minimum level of reward could be determined by incorporating risk management methodologies such as value at risk or profit at risk. Understanding how to balance the cost of new information with its benefits is an important next step.

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