HOW PREDICTABLE IS SPACE EXPLORATION?

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HOW PREDICTABLE IS SPACE EXPLORATION? by Michael Christopher Howell, May 2020

ABSTRACT

Recent research has suggested that progress in technological domains often improves in an exponential manner. The traditional method to model this increase in capability has been to fit a model with time or effort as an independent variable and then extrapolate the trend (Moore, 1965; Wright, 1936; Magee et al., 2014). Although other methods have been used as well (Nagy et al, 2013). While effective, these methods are not the only way to forecast such trends and they have their own limitations. Recent research has indicated that a potentially useful approach for modeling technological improvement is time series analysis.

I use this approach to build on previous research which suggests that space exploration technology displays an exponential trend. This trend can be measured with the metric of spacecraft lifespan. Time series models of this improvement will be shown along with forecasts for future improvement. Finally, further directions for future research will be explored.

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Background

It has long been recognized that technology is the primary driver of sustained economic progress. However, the recognition that technology progresses in a predictable way is a relatively recent phenomenon. Some of the earliest work in this area was conducted by the engineer Theodore Paul Wright (Wright, 1936). Wright described a phenomenon he observed while supervising the production of aircraft. As the batch size of a model of aircraft increased, the per-unit cost to manufacture those aircraft decreased at a predictable rate. The approximate relationship was a 20% drop in cost for every doubling of production volume. This phenomenon has been attributed to "learning by doing" where productivity is improved through the accumulation of experience. Subsequent research indicated that this pattern holds for a variety of industries although the rate of cost decline varies (Hax & Majluf, 1982). This phenomenon has been referred to by various names such as learning curves, experience curves, and Henderson's law (Wikipedia, 2019). Contemporary research into technology foresight uses the term Wright's law, so in this paper we will be using this term.

A much better-known trend is Moore's law. Originally this phenomenon was described by one of the co-founders of Intel, Gordon Moore, in 1965. Moore noted a regular doubling of the number of components that could be built into an integrated circuit (Moore, 1965). This trend has continued since then with an approximate doubling occurring every 18 months to two years. While this is a well-known trend, what is often overlooked is that this applies to many other fields outside of computing. Figures 1-7 illustrate examples of exponential trends found in other fields. The purpose of these figures is to show evidence that exponential trends in technological progress have existed, not necessarily to track those trends into the present. For this reason, some of these figures will be lacking data from recent years. As will be explained below, both Wright's law and Moore's law have their strengths, and the question of which to use is a subject of ongoing research.

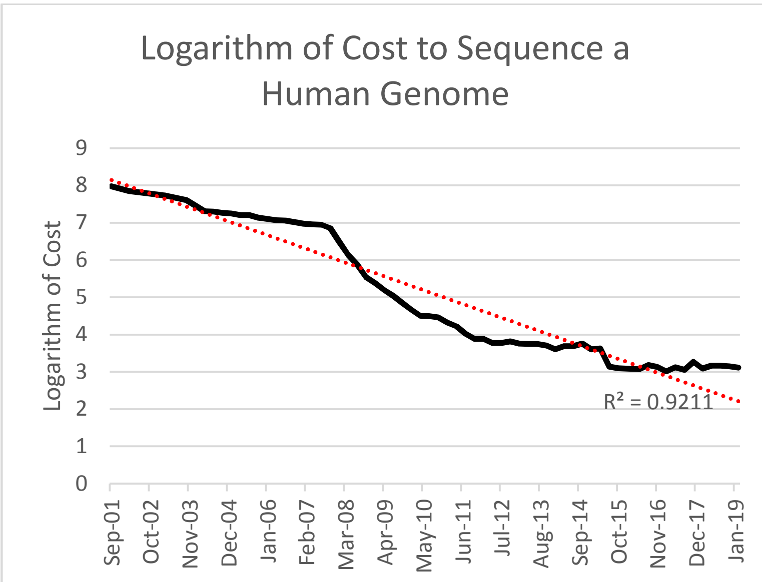

Figure1- Cost to sequence a human genome (Source: https://www.genome.gov/sites/default/files/media/files/2019-06/Sequencing_Cost_Data_Table_Feb2019.xls)

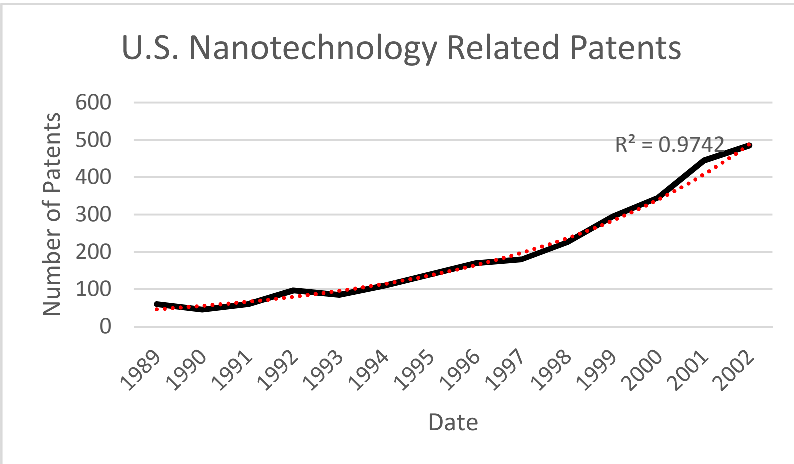

Figure 2 - Nanotechnology related patents (Source: ETC Group, "From Genomes to Atoms: The Big Down," p. 84, http://www.etcgroup.org/documents/TheBigDown.pdf.)

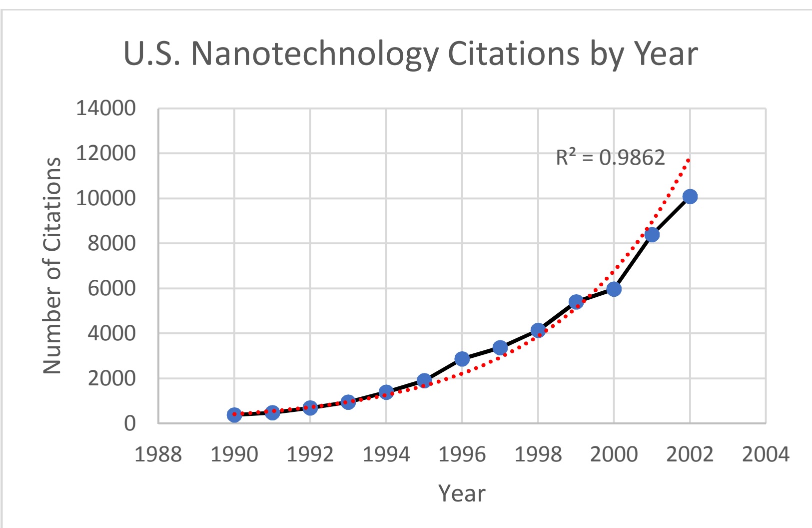

Figure 3 - U.S. nanotechnology citations by year (Source: ETC Group, "From Genomes to Atoms: The Big Down," p. 83, http://www.etcgroup.org/documents/TheBigDown.pdf.)

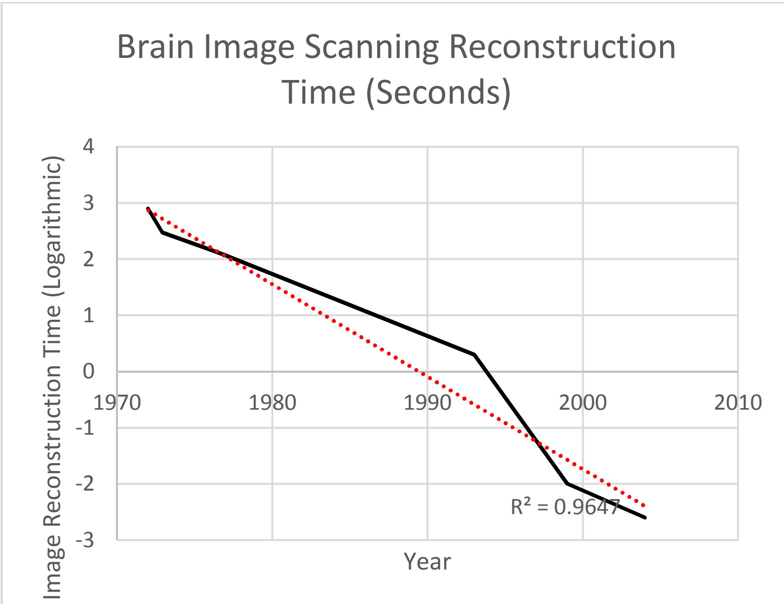

Figure 4 - Brain Image Scanning Reconstruction Time (Seconds) Source: Manuel Trajtenberg, Economic Analysis of Product Innovation: The Case of CT Scanners (Cambridge,Mass.:Harvard University Press, 1990); Michael H. Friebe, Ph.D., president, CEO, NEUROMED GmbH; Philips Medical Systems www.medical.philips.com; Magnetic Resonance Technology Information Portal, http://www.mr-tip.com/serv1.php?type=db1&dbs=brain%20imaging&set=4.

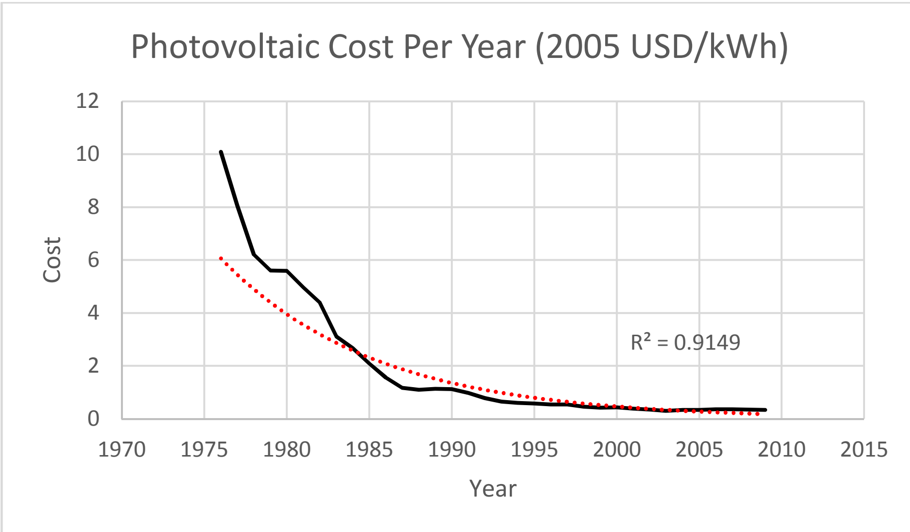

Figure 5 - Photovoltaic cost per year (2005 USD/kWh). Source: Beyond the learning curve: factors influencing cost reductions in photovoltaics by Nemet, G.F. (2006)

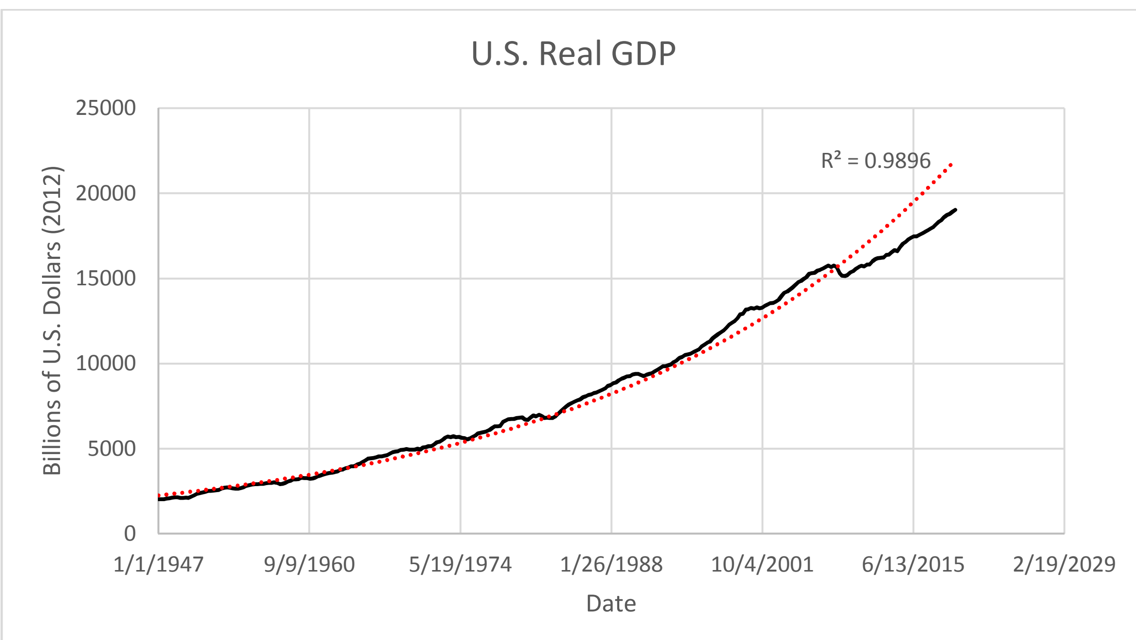

Figure 6 - U.S. Real GDP. Source: https://fred.stlouisfed.org/series/GDPC1

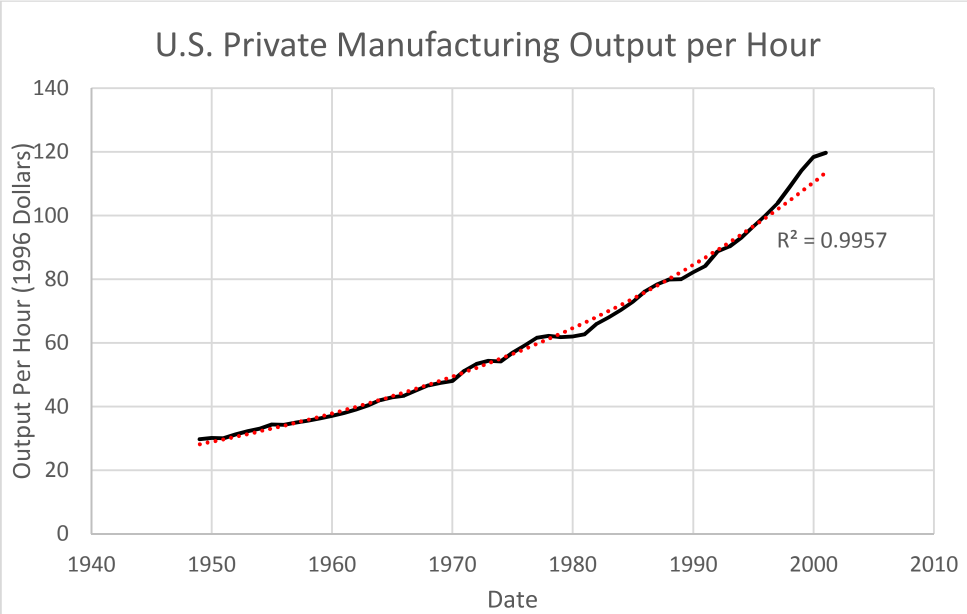

Figure 7 - U.S. private manufacturing output per hour. Source: Bureau of Labor Statistics,Major Sector Multifactor Productivity Index, Manufacturing Sector: Output per Hour All Persons (1996 = 100), [*http://data.bls.gov/PDQ/outside.jsp?survey=mp*](http://data.bls.gov/PDQ/outside.jsp?survey=mp) *(requires JavaScript: select "Manufacturing," "Output Per Hour All Persons," and starting year 1949)*

Theoretical Framework

Research into why technological capability progresses the way it does is ongoing, but several scholars have provided valuable insights. Traditionally, economists and management scientists assumed technological progress as exogenous variables (Solow, 1957). However, others have contested this view since it leaves technological progress itself unexplained. The work of (Arrow, 1962) and (Romer, 1990) have given rise to Endogenous Growth Theory which seeks to understand technological progress as a result of market forces. Romer contends that the economic properties of knowledge, namely nonrivalry and partial excludability, provide "spillover effects" and allow others to benefit from a firm's private research.

This view that markets incentivize technological progress finds confirmation in the work of Sokoloff, (1988) who demonstrated evidence of an increase in patent activity in the early United States coinciding with business cycles, access to domestic markets, and disruptions in international markets. Presumably the disruption in international markets increased opportunities for domestic producers. This increase in patent activity seems to have been due to technical workers who became interested in innovation in response to economic incentives (Sokoloff & Khan, 1989). Besides these important economic factors, in order to understand why technology progresses the way it does we can also consider the invention process itself. The invention process is important because invention springs from scientific knowledge, but knowledge must be codified into specific designs in order to develop technologies. The noted polymath Herbert Simon in *The Sciences of the Artificial* argued that design had its own logic comparable to natural science and was worthy of independent study, even going so far as to lay out a potential theoretical foundation for a "science of design" (Simon, 1969).

Research into design has generated many models of how invention occurs. Vannevar Bush, the head of the Office of Scientific Research and Development during WWII argued for the importance of basic, publicly financed, scientific research useful for technological development (Bush, 1945). The Function-Behavior-Structure (FBS) model conceptualizes design as consisting of transformations between what is required of an object (Function), what an object is (Structure), and what an object does (Behavior) (Gerro & Kannengiesser, 2014) . The C-K model contends that traditional design theories are too tied to a particular domain and cannot account for innovation. Therefore, it focuses more on describing the development of new knowledge and concepts (Hatchuel & Weil, 2003).

However, one theory that has had some noteworthy empirical validation and seems able to describe exponential technological change is Analogic Transfer (Basnet & Magee, 2016). This model views the recombination of existing scientific and engineering concepts as a primary driver of technological progress. In the context of this theory, we can think of a technological domain in terms of base domain and a target domain with a similar structure as the base. We can then think of progress occurring when the inventor applies ideas from the base to the target domain. Magee and Basnet describe these ideas as consisting of engineering principles known as Individual Operating Ideas (IOI) and scientific concepts known as Units of Understanding (UOU).

Intuitively, we begin to see how this may lead to accelerating progress. As more knowledge is acquired, more combinations of ideas become possible, and the stock of knowledge should grow faster. Of course, this process cannot continue forever since there are only so many applications for an individual idea. But it seems reasonable to think that many ideas have the potential for many applications. Magee further demonstrates that simulations conducted with these assumptions show an exponential growth in the number of concepts available for reapplication which is further consistent with observed exponential trends for a variety of technologies. Figure 8 shows the results of one of these simulations.

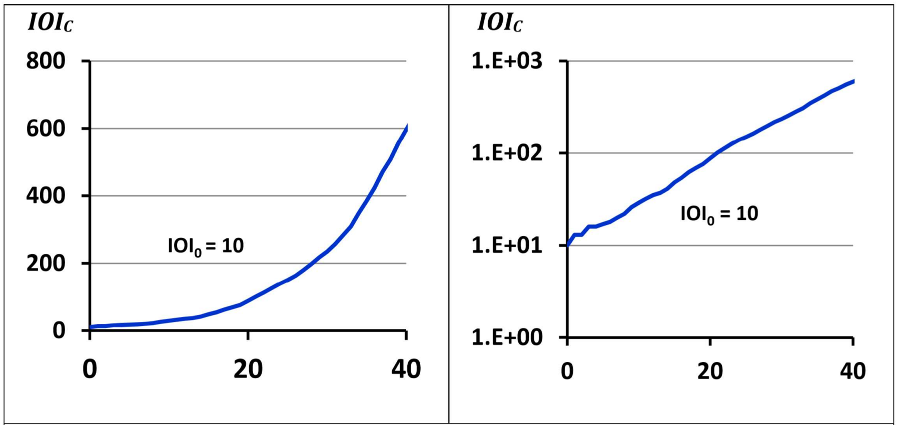

Figure 8- Simulation of idea recombination. Source: Modeling of technological performance trends using design theory by Magee, C.L. and Basnet, S. (2016)

Returning to the subject of design, the notion of artifact interactions, introduced again by Herbert Simon, provides a possible explanation for the observed differences in rates of progress for various technologies. Before we consider artifact interactions, it is necessary to think more deeply about the economic and functional properties of designs. Using a framework developed by Baldwin and Clark (2006) we can begin to think of design in terms more fundamental than individual technologies. Designs create economic value by providing a stream of future benefits from the creation of a given solution which solves a particular problem. Therefore, we may think of designs as an asset class since they have no intrinsic value in themselves and only derive value from what we can do with them. Furthermore, since a design consists of an option and not a requirement to create the solution, we could potentially assign value to them using option valuation techniques. The chief method is the "Net Option Value" method which represents the value of the system as the sum of the value of the minimal system and the values of each module. It should be noted that Robert Merton's work on option valuation used widely in finance does not apply here since there is no underlying financial asset (Merton, 1973). Continuing with

Baldwin and Clark's framework, we can think of design as consisting of decisions about how to manipulate artifact components as well as interdependencies among these decisions. These decisions may be categorized based on how a change in one component may require a change in one or more other components. These decisions may be grouped into three categories:

1.) Independent – Components in one domain may be changed without affecting components in another domain.
2.) Integral – Changes in one domain require changes in another domain.
3.) Modular – Components are independent but operate on a similar set of design rules.

These decisions can be visualized using a graph known as a "Design Structure Matrix" or DSM. A DSM simply consists of a 2-dimensional matrix of all the system components with an "x" placed in each cell that contains a dependency (Wikipedia, 2019).

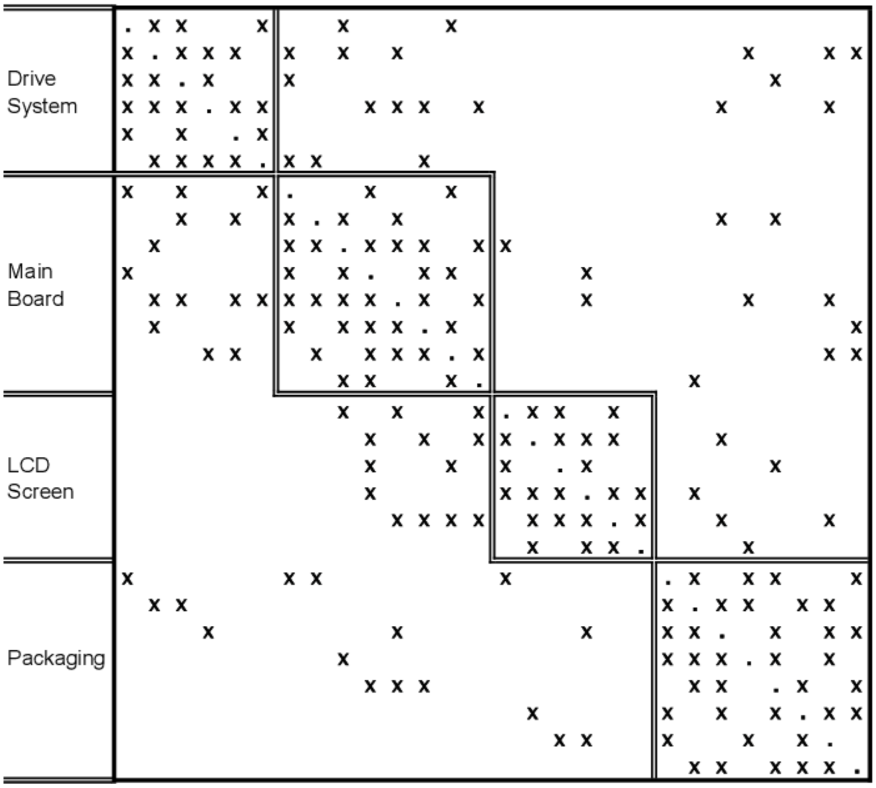

Figure 9 - Example of Design Structure Matrix (DSM). Source: McCord, Kent R., and Steven D. Eppinger (1993). "Managing the Iteration Problem in Concurrent Engineering." Working Paper 3594-93-MSA, MIT, Cambridge, MA (August).

For technologies with a high degree of integral design interactions, improvements may be more difficult as more design time must be spent accommodating changes in the entire system instead of a single component as is the case with more modular designs(Basnet & Magee, 2016). McNerny et al. (2011) has provided empirical evidence that this may explain the observed differing rates of technological progress in various domains.

Comparison of various models

Wright's law and Moore's law both successfully model increases in technological performance. Therefore, it's only natural to wonder how the two models compare. Besides the technologies we demonstrated earlier, Magee (2014) demonstrates the general applicability of Moore's law in 28 technological domains while Nagy et al. (2013) demonstrates the relationship with 62 technologies. Yet despite the evidence of these trends, some may be skeptical of the use of time as an independent variable. To quote Kenneth Arrow (1962):

" *…trend projections, however necessary they may be in practice, are basically a confession of ignorance, and, what is worse from a practical viewpoint, are not policy variables".*

Common sense tells us that technological progress must have its root in human effort. We may observe an approximate doubling in processor capacity every 18 months, but if we were to stop all chip research and development, we know there would be no additional progress. As mentioned earlier, the consensus of modern economics is that technological progress is endogenous to economic activity. These facts may make us hesitant to use time as an independent variable, but perhaps they shouldn't. While time may not directly capture the impact of effort it may indirectly capture it by serving as a proxy if whatever is driving technological progress is occurring in a time dependent manner.

Since it is effort that must drive technological progress, it would be natural to assume that Wright's law would be the better model since this relates cumulative production to technical performance. However, there are some important caveats to consider. Wright's original paper studied cost reductions for a particular model of airplane in a particular factory, while contemporary research typically uses cumulative production in an entire industry. This may lead us to conclude that Wright's law isn't necessarily capturing the accumulation of tacit knowledge as indicated in Wright's original paper but is serving as a proxy for general effort. An empirical study comparing the two models has shown that while there is evidence that Wright's law does model technological progress very slightly better than Moore's law, the differences are not great (Nagy et al., 2013). Viewing technological progress as an outcome of "learning by doing" also ignores the fact that technology advances primarily through experimentation and research and not just repeatedly producing the same design. In fact, there does seem to be some evidence that the use of research-based effort as an independent variable may be an effective way to predict technological improvement (Benson & Magee, 2015) (Foster, 1985). There is also the issue of correlations among effort variables since production is correlated with revenue and revenue is correlated with R&D expenditures and patents (Magee et al., 2014).

If, however, learning-by-doing does play an important role in technological development, then learning curves also have an important predictive limitation due to product life cycles. Discoveries by Leontief (1953) demonstrated that the United States primarily exported labor intensive goods despite being the most capital rich country in the world – a phenomenon that defied current economic theory. This has been attributed to life cycles for manufactured products, and by extensions to other technologies. Ayers (1992) points out that the life cycle influences how learning can take place and thus could interfere with predicted improvements. In the early stages of a technology, there is not much standardization of production and therefore learning is still important. However, in later stages much of the knowledge is now embodied in machines and processes. In this stage of the product life cycle, observed improvements have often been observed to plateau, as was the case with automobiles. One cannot predict beforehand when a product will reach a particular life cycle stage so production volume may not be as useful as time for making predictions.

These two models may be equivalent under the right circumstances. Sahal (1979) has demonstrated that Wright's law is identical to Moore's law when production increases exponentially. Subsequent research has shown that this relationship holds for any effort variable and not just production (Magee et al., 2014). However, such a trend in production has been observed by Nagy (2013) for many technologies and seems reasonable for commercial technologies given that any improvement in technology should lead to an increase in demand if elasticity of demand is held constant. Because of this, many would assume that exponentially increasing production is the cause of exponential improvements in technology. As reasonable as this sounds, empirical evidence shows that exponential improvements often occur when production is not increasing exponentially (Magee et al., 2014). There are even scenarios when technologies improve significantly without much commercial production at all (Funk & Magee, 2015). Based on all the available evidence, it would seem reasonable to conclude that Moore's law may be a better description of how technology evolves.

Implications of exponential change

The exponential trend observable in so many technological domains has profound policy implications which seem to be largely unappreciated by society. A full discussion of these implications is beyond the scope of this paper, but some examples may help to illustrate this point. For economics, Solow (1957) and Romer (1990) have pointed to the importance of scientific and technological development to economic growth. If one assumes that technological progress is exponential, then one should expect enormous economic growth in the near future. As of 2016, the World Bank gave the U.S. a Gini Coefficient of 41.5 which is quite high for the developed world (Wikipedia, 2019). If this value continues to hold then this would imply vast wealth inequalities which may serve to undermine democratic institutions. Furthermore, Kartik Gada, an investment banker specializing in AI companies, has argued that the existence of exponential technological change implies deflationary pressures in the economy similar to what occurs with computer technology (Gada, 2019). If true, this may imply the need for greater quantitative easing from central banks than is currently practiced.

Risk assessment is an important area to consider as well. Nick Bostrom (2013) has argued that emerging technologies create new "existential risks" such as synthetic viruses, molecular nanotechnology, and uncontrolled artificial intelligence. Humanity has a long history of surviving natural risks but unlike most natural risks these may have the potential to eliminate humanity. A problem with these risks is that they are generally not taken very seriously, a fact which may have its roots in certain cognitive biases. Probability estimates of events are inevitably skewed by an individual's ease of recalling similar events (Lichtenstein et al., 1978). Considering this bias and the fact that human extinction has never occurred, it is likely that humans underestimate its likelihood (Yudkowsky, 2008). Bostrom along with Anders Sandberg have suggested that increasingly sophisticated neuroscience and biotechnology could enable widespread cognitive enhancement. This has the potential to change the role of medicine and government regulation of medicine (Bostrom & Sandberg, 2009). As for the environment, there is evidence which suggests that solar energy technology is likely to continue to drop in price by 10% per year, while the prices for coal and nuclear energy have remained largely unchanged (Farmer & Lafond, 2016). Wright's law would suggest that aggressively subsidizing the installation of solar panels would improve their performance and impact climate change more quickly than most realize.

It must be emphasized that we are not necessarily endorsing or criticizing any of these views. We list these scenarios only to illustrate the importance of technology foresight to society. Technology forms the basis for civilization, and by understanding how it may develop we can more effectively bring about futures we prefer.

Exponential space travel

While these exponential trends have been observed for several technologies, one area that it has not been sufficiently noticed for is space travel. In fact, there is a widespread perception that progress in space travel has mostly stalled (Hicks, 2015). We believe this view is mistaken and our currently ongoing research suggests such a trend exists (Berleant et al., 2017) (Berleant et al., 2019, Appendix C).

Currently, the most popular method to model trends in technological progress is to use simple linear regression to extrapolate a trend with time as the independent variable. While this method has worked quite well in the past, it has a problem in this domain. This problem occurs when product lifespan is used as the attribute which is hypothesized to undergo an exponential increase over time..

Problems with curve fitting

If one assumes an exponential increase in spacecraft lifespan over time, then certain problems can be easily demonstrated with any regression models that are built. One way of measuring spacecraft lifespan is to subtract the time of launch from the time of mission termination, and look at average lifespan for craft launched in a given year.

This causes a problem with measuring mission durations of spacecraft launched in recent years. Since only short-lived spacecraft could be measured for recent years, data from those launch years will naturally be biased toward shorter lifespans since longer missions have not yet ended. Another approach is to measure lifespans in terms of the year the spacecraft "died". While this avoids the bias problem, it creates certain points where the model breaks down. The problem comes from the fact that time increases linearly with time whereas lifespan increases exponentially. At a certain year, mission lifespan will be growing faster than time passes. Eventually this problem gets so bad that the average lifespan predicted for craft whose lifespans end in a given year will be greater than the entire period that space technology has existed (Appendix B, Howell et al., 2019).

However, there are ways around these problems. All of the deep space mission data are indexed by time, and therefore we should be able to use existing time series methods to make predictions for future values of spacecraft lifespan.

This approach has several advantages. First, we need to consider the issue of how limited the data are. Because the data are limited it is difficult to build casual models relating an independent variable (i.e. cost, R&D funding, etc.) to technological progress. Even if the data weren't limited there are the previously mentioned problems of how to properly define the appropriate independent variable to model causality. With time series modeling we do not have to make any guesses about what variable might be causing the technological progress.

Second, if one models lifespan increase this way, this should overcome the previously mentioned bias problem since current values for the time series are defined in terms of previous values as will be explained below. We can build predictive models based on the data that we can be sure are free of that type of bias.

The third advantage is more subtle and one not often appreciated by futurists. Time series analysis allows us to define a distribution of forecast probabilities and decide how confident we should be in our predictions. All forecasts are wrong, but if we can define the *way* that they will be wrong we can make more intelligent predictions. As Farmer and Lafond (2016) write:

"Point forecasts are of limited value unless they are very accurate, and when uncertainties are large they can even be dangerous if they are taken too seriously. At the very least one needs error bars, or better yet, a distributional forecast, estimating the likelihood of different future outcomes."

Box Jenkins Method

Fortunately, there is a well-defined way to make distributional forecasts that can be applied to this problem. The Box-Jenkins method is a method for modeling a time series developed by the statisticians George Box and Gwilym Jenkins in 1970 (Box & Jenkins, 1970). The method models a stochastic process by fitting a model to a dataset using the following stages: model specification, parameter estimation, and model checking (Wikipedia, 2019).

Model Specification

Autoregressive and Moving Average Models

The Box Jenkins methods uses two categories of models which together can describe many time series. The first are Autoregressive models, which consist of a linear combination of previous values of the time series (Pankratz, 1983, 47-50). As the name implies, the forecast for the time series is a regression on itself. Autoregressive models take the form:

$$y_t = C + \phi_1 y_{t-1} + \phi_2 y_{t-2} \ldots + \phi_p y_{t-p+} \varepsilon \sim N(o, \sigma^2) \quad (1)$$

Where y_t is the current value of the time series, C is a constant, and ε is an i.i.d. process with a mean of zero and a constant variance (Ibid.). The subscript p signifies the number of previous values of the time series that is considered (Ibid.). For this reason, an autoregressive model can be identified as AR(p). Each previous period is weighted with a corresponding parameter ϕ_p.

The second categories of models are the Moving Average models. Rather than describing the current value of the time series as a linear combination of previous values, Moving Average models describe the series in terms of previous residuals. Much like the Autoregressive model, the Moving Average model is identified as MA(q) where q is the number of previous residuals considered (Ibid.). If a_t is the residual associated with period t then a Moving Average model of order q is defined as:

$$y_t = \mu + \theta_1 a_{t-1} + \theta_2 y a_{t-2} \dots + \theta_q a_{t-q+} \epsilon_t \sim N(o, \sigma^2) \tag{2}$$

These two models may be combined to expand the behavior that can be modeled. Such models are denoted as ARIMA(p,d,q). Where *p* signifies the autoregressive portion, *q* represents moving average portion, and *d* represents the number of differencing operations. The differencing operation is sometimes referred to as integrating and represents the “I” in the ARIMA specification (Ibid..,95-96). Differencing will be described in more detail below.

The constant C in equation (1) is closely related to the average change in the time series from one observation to the next. For any stationary ARIMA(p,d,q) this is equal to:

$$C = \mu(1 - \sum_{i=1}^{p} \phi_i) \tag{3}$$

It should be noted that in the case of a pure MA model the constant C reduces to the mean of the differenced series μ. This is because there are no autoregressive parameters ϕ_i(Ibid..,101-102).

Stationarity and Differencing

Before the statistical properties of any time variant data are analyzed, it must be confirmed that those properties are stable. This is determined in the model identification stage. More specifically, we want to determine if the data has a stable mean and variance. If these properties are stable, then the series is said to be stationary (Vandaele, 1983., 12-16).

But what if these properties do change over time? A series with a nonstationary mean can usually be identified by a visible trend in a particular direction, either upward or downward.

Unfortunately, it is not easy to tell if the trend is the result of an underlying deterministic process or if the trend is due to chance. If the trend is a function of time (i.e. deterministic) it is said to be trend stationary, since once the trend is accounted for we are left with a stationary process (Nielsen, 2005). An important property of trend stationary processes is that they display mean reversion and thus all random shocks are transitory (Ibid.).

A difference stationary process is different in that random shocks affect the process permanently and no mean reversion occurs (Ibid.). Sometimes deviations from the trend are due to chance but there is an underlying deterministic trend. From (4), μ would be the underlying trend.. Such a process is modeled by a random walk taking the form:

$$y_t = y_{t-1} + \varepsilon \sim N(o, \sigma^2) \tag{4}$$

where y_t is a time series and ε is an i.i.d. stochastic process with a mean of zero and a constant variance (Hurvich, 2019). The future values of a random walk are completely determined by their current position plus a noise term and are examples of Markovian processes i.e. processes without any memory (Weisstein, 2019). A real world example of such a process would be the stock market since the current price of the stock is unaffected by its previous values (Malkiel, 1973). A difference stationary process can be identified informally by examining a plot of the autocorrelation of the series at various lags. If the autocorrelation decreases very slowly then this is a sign of a stochastic trend (Vandaele, 1983, 66-72). A difference stationary process can also be identified by the autocorrelation function of the first differences of the series. Recall that (4) identifies a nonstationary process as a random walk. Since the change from one period to the next is random then the autocorrelation function of the first differences should be uncorrelated white noise (Hyndman & Athanasopoulos, 2019).

Another, more formal, way is to use a Dickey-Fuller test. Equation (4) defines a nonstationary process as a random walk. These processes are a special case of an AR(1) process where ϕ =1. Therefore, determining if an autoregressive process is a random walk amounts to determining if ϕ =1. To see how we can determine if a process is a random walk, we start with an AR(1) process:

$$y_t = \phi y_{t-1} + \varepsilon \tag{5}$$

If we were to describe the change in one period to the next, we can subtract the previous values of the time series to get:

$$y_t - y_{t-1} = \phi y_{t-1} + \varepsilon - y_{t-1} \tag{6}$$

Which can also be written as:

$$y_t - y_{t-1} = (\phi - 1)y_{t-1} + \varepsilon \tag{7}$$

If we replace $(\phi - 1)$ with β and $y_t - y_{t-1}$ with Δy_t ,we are left with:

$$\Delta y_t = \beta y_{t-1} + \varepsilon \tag{8}$$

It is easy to see that (8) is simply a linear regression equation. The hypothesis of a unit root (i.e. $\phi = 1$) is equivalent to saying $\beta = 0$. Therefore, we can create the following hypothesis test for a unit root with:

$$\begin{aligned} &H_{0:}\, \beta = 0 \\ &H_{A:}\, \beta < 0 \end{aligned} \tag{9}$$

The p-value of our estimate of β can be compared with the null hypothesis of nonstationarity which we can then accept or reject (Dickey & Fuller, 1979). This model can be extended using multiple regression coefficients to accommodate AR processes greater than 1 (Zainotz, 2019).

So, by examining the plots of the autocorrelation and partial autocorrelation coefficients as well as the use of a Dickey Fuller test we can reasonably conclude if the data is nonstationary to the desired p-value.. If the data is nonstationary, then the model can be rendered stationary by subtracting each time series value from the one preceding it, a process known as differencing (Pankratz, 1983, 24-28). It must be emphasized that only data with evidence of a stochastic trend should be differenced. Differencing a series with deterministic trend can introduce spurious patterns into the data and complicate analysis (Ibid., 166). In the case of deterministic trend, the data can be made stationary by explicitly including the trend in the model.

Determining Model Type

After the stationarity of the data is confirmed, the next step is to determine if it is an Autoregressive or Moving Average process. This is done by again examining plots of the autocorrelation and partial autocorrelation of the data. Autoregressive processes are characterized by a gradual decline of the autocorrelation function and Moving Averages are characterized by a sudden cut off of the autocorrelation function (Ibid.., 121-124).

Once it is decided if the process is a Moving Average or Autoregressive it is now time to determine how many terms to include. For autoregressive processes, the number of terms to use is equal to the number of lags with a partial autocorrelation statistically different from zero. For Moving Average processes, the number of terms to use is equal to the number of lags of the autocorrelation function statistically different from zero (Ibid.).

Parameter Estimation

In the parameter estimation stage, maximum likelihood estimation is used to determine the appropriate value of the parameters to use. Maximum likelihood estimation (MLE) finds the values of the parameters which maximize the probability of obtaining the observed data and was the method preferred by Box and Jenkins (Ibid.., 193). However, another method that can be used is least squares estimation. Least squares estimation finds the value of the parameters that minimizes the sum of the squared residuals. It can be shown that if the residuals are normally distributed, then this method is equivalent to MLE (Ibid.). For this reason, it is most often used by contemporary practitioners for estimating ARIMA models.

Model Checking

Finally, In the model checking stage, the model is tested to confirm its integrity. Parsimony is emphasized in order to avoid overfitting. This is usually measured by the Akaike Information Criterion (AIC) (Akaike, 1974) or the Bayesian Information Criterion (BIC) (Schwartz, 1978). In both cases, models having lower values of each are preferred. The residuals of the model are then checked to ensure that they are uncorrelated and have a constant mean and variance (Pankratz, 1983, 224-225).

ARIMA Error Models

ARIMA models are based on the assumption of uncorrelated error terms. However, sometimes it may be useful to allow correlation in the error terms. The method is relatively straightforward. A simple linear regression is created using an exogenous variable and an ARIMA model is fitted to the resulting errors. They can be expressed as:

$$y_t = \beta x_t + n_t \tag{10}$$

where β is a parameter, x_t is a exogenous variable, and n_t is an ARIMA error process. An important consideration for these models is the future value of x_t . When the exogenous variable is time then the future values are straightforward. However if x_t is some other variable then it must be forecast and the resulting predictions combined with the ARIMA error forecast (Hyndman & Athanasopoulos, 2019).

Forecast Distribution

Forecasts for the future of a time series have important properties, so studying them will be useful in understanding the range of possibilities for the future of the series and how confident we can be in our predictions. Let *t* be the current time point and y_t equal the current value of a stochastic process *y* which we will refer to as the origin. Next, let all previous observations of our process be denoted by *I*. Furthermore, let *f* equal our forecast horizon where *f > 0* . Therefore, our forecast for an ARIMA (p,d,q) process will be denoted by the conditional expectation:

$$\begin{aligned} y_t(f) &= E\left(y_f \middle| I\right) \\ &= \mu(1 - \textstyle\sum_{k=1}^{p} \phi_k) + \phi_1 y_{t-1} + \theta_1 a_{t-1} + \phi_2 y_{t-2} \\ &+ \theta_2 a_{t-2} \ldots + \phi_q y_{t-q} + \theta_q a_{t-q} \end{aligned} \tag{11}$$

Each random shock a_{t-q} is estimated from the residual of that period and the expectation of this shock is zero for $f > q$ since it is a random variable (Pankratz, 1983, 240-250).

According to Hyndman (2014), all time series predictions are affected by four sources of uncertainty. These sources are the assumption of the continuation of past trends, the quality of the model, parameter uncertainty, and random shocks.

The assumption of continuation of past trends is required for making any forecast so it must just be accepted. Model quality can be optimized by considering model fit and distribution of the residuals. Parameter uncertainty can theoretically be minimized using simulations, but this vastly increases the complexity of making predictions(Hyndman, 2014). For these reasons, the only source of uncertainty that will be considered for making prediction intervals will be the error term. Starting with (1) and assuming a normally distributed error term with constant variance, the prediction interval can be specified as:

$$y_t(f) \ \pm c\sigma_f \tag{12}$$

where $y_t(f)$ is the point forecast, σ_f is the standard deviation of the forecast and c is the coverage probability (Hyndman & Athanasopoulos, 2019). In practice, σ_f is the estimate of the standard deviation of the residuals and c depends on the desired significance level of the prediction. Most ARIMA practitioners use 80% and 95% prediction intervals which correspond to values of c equal to 1.44 and 1.96 respectively (Hyndman, 2014). So if we were to use these values then we could be reasonably confident of where 80% and 95% of the actual values would fall.

If parameter uncertainty is to be considered, then there are certain points worth keeping in mind. Sampson (1991) has argued that with parameter uncertainty forecast variances grow with the square of the forecast horizon but only linearly with no parameter uncertainty. Therefore, according to Sampson, parameter uncertainty might be a greater source of forecast variation than random errors. Doyne Farmer and Francois LaFond both agree with this view in their own studies on the use of time series analysis for forecasting technological change. (Farmer

& Lafond, 2016)(LaFond et al., 2018). While Clements (2001) agrees that parameter uncertainty can be great, he also points out that this assumes that the sample size remains constant as the forecast horizon expands. If the sample size is allowed to increase, then the result is similar to that of a forecast without parameter uncertainty.

These are important points to keep in mind. However, to simplify matters we have decided to use the more orthodox approach originally recommended by Box and Jenkins. Farmer and Lafond's studies provide an important resource for showing how technological progress can be modeled using time series analysis, so it may be worthwhile to show how this method differs from theirs. They assume that the technologies follow a random walk pattern and thus have unit roots. Many of their time series that were studied were too small for unit root tests to be effective but nevertheless their forecasts were consistent with the hypothesis of a unit root. The spacecraft lifespan data studied in this paper show no signs of a unit root and in fact show signs of trend stationarity as will be explained below.

A great deal of their analysis was also focused around the nature of their data, which consisted of numerous technology time series of different lengths and different rates of progress, many of which were too short to form proper ARIMA models. The data often consisted of averages taken over time. When data is averaged like this it can cause spurious correlations that can distort statistical analysis (Working, 1960). This false correlation had to be accommodated in the prediction intervals. The spacecraft lifespan data do not contain the same issues as the data studied by Farmer and Lafond so their exact method may not be necessary for forming predictions, although it may still be effective. Finally, their method directly incorporates parameter uncertainty and experimentally derives prediction intervals whereas the Box-Jenkins method derives prediction intervals through the estimate of the variance of the residuals.

Data

The data were manually curated from various Wikipedia sources. In many cases the relevant Wikipedia articles were edited to contain the data points so that automated harvesting of data points from Wikipedia would work (Berleant, 2019).. The values of interest are the launch date, death date, name of mission, order of launch, and the country of origin. At the time of this writing the data are stored in a publicly accessible google spreadsheet located at https://docs.google.com/spreadsheets/d/1ZtfkjbcTOoZTbETUkOY5Hlq5SY5GREvFYjgzmKZQww4/edit#gid=1036188494.

Methods and Results

All analysis was conducted using various R statistical packages. To begin, we can examine whether or not the data may be stationary. Figure 11 displays a plot of the Autocorrelation of the data which decays very slowly and may be an indication of stochastic trend. After taking first differences a plot of the ACF is shown in Figure 12. The ACF plot of the differences displays significant autocorrelation and, as explained earlier, is inconsistent with nonstationary data. There is an autocorrelation of -0.5 at lag 1 which drops off immediately to almost zero, which is a classic sign of over differencing and a further indication that the original data may be trend stationary (Nau, 2019). Finally, Figure 10 shows an Augmented Dickey Fuller test with a null hypothesis of nonstationarity demonstrates statistical significance and is again consistent with stationarity. The model in this test incorporates a constant and a linear trend, so subsequently generated models should incorporate both of these parameters as will be demonstrated below. Considering all of this evidence, we can conclude that this data is stationary and doesn't require differencing.

A plot of the Partial Autocorrelation Function of the data is shown in Figure 13. The series displays significant correlations at lags 1,2, and 5. The gradual decay of the ACF combined with the significant partial correlations at lags 1 & 2 that cut off abruptly seem to indicate that an AR(2) model may be most appropriate. This makes intuitive sense, since it would seem that if a technology is improving then its current capability must be correlated with its previous capability. The significance at lag 5 cannot be readily explained from the data. One possibility is that NASA must take the position of planets into account for longer missions. Therefore, longer missions appear semi-regularly.

```
        Augmented Dickey-Fuller Test

data:  tsdata
Dickey-Fuller = -3.8434, Lag order = 4, p-value = 0.01934
alternative hypothesis: stationary
```

Figure 10 - R Output of Augmented Dickey-Fuller Test

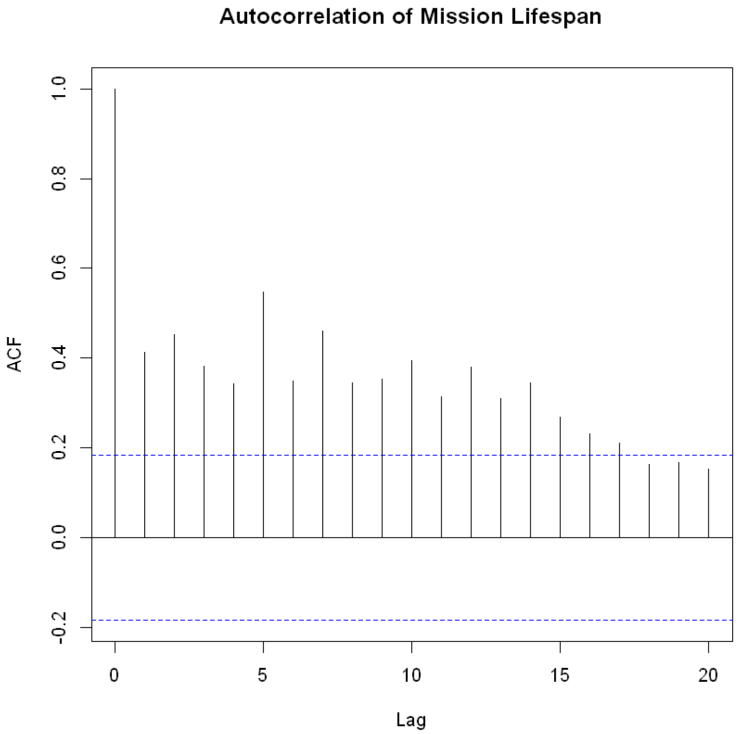

Figure 11- Autocorrelation of Spacecraft Lifespan

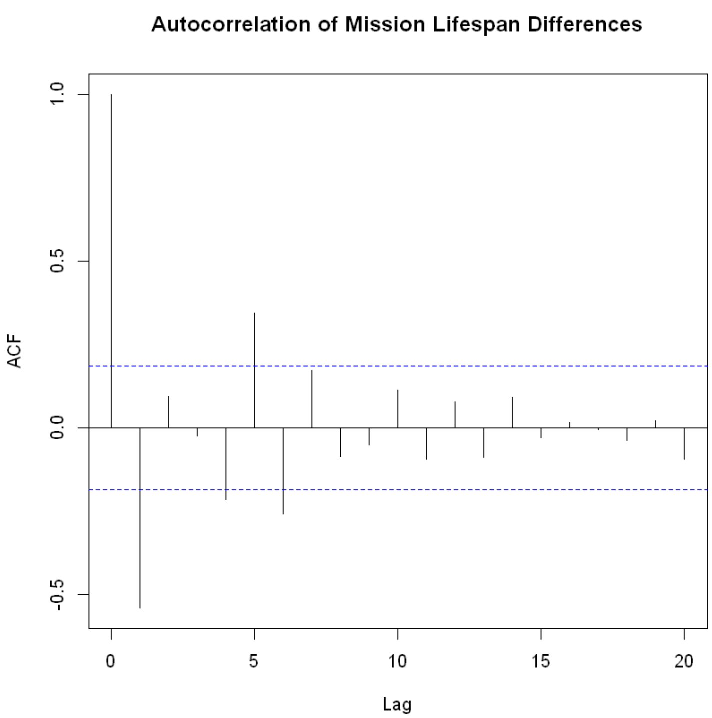

Figure 12- Autocorrelation of First Differences

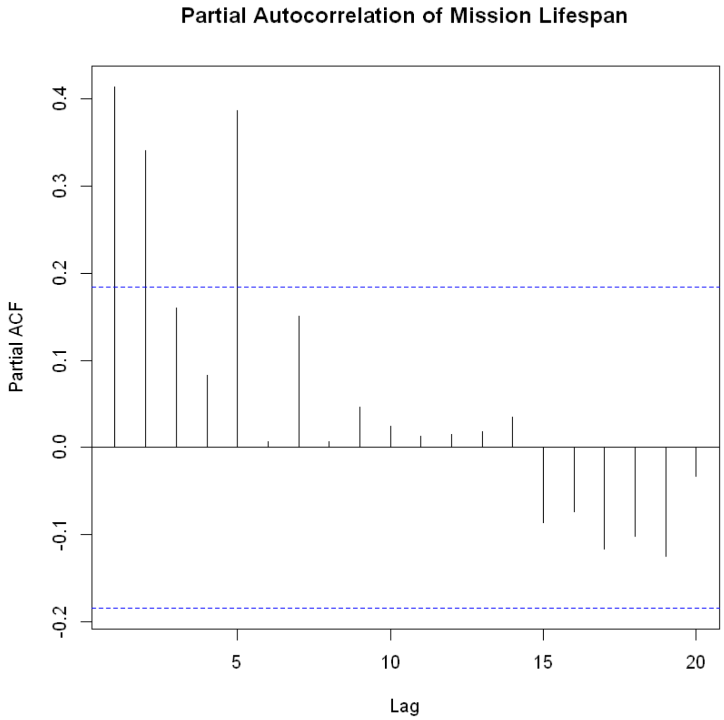

Figure 13- Plot of Partial Autocorrelation Function

As stated earlier, it is usually desirable to choose a model with the lowest AIC and BIC values since these values measure the parsimony of the model. However, several models were discovered which were largely consistent with autocorrelation trends observed in the data. In addition, these models all had negligibly different AIC and BIC values. Therefore, each of these models will be examined, forecasts will be produced, and the residuals examined to determine model quality.

The accuracy of the forecasts will be checked by Hindcasting, that is comparing forecasts with actual values. The forecasts will also be evaluated by computing the mean squared error of the forecasts. This will be done by forming models based on 85 percent of the data and the remaining 15 percent will be compared with the model's predictions. As was mentioned earlier, the Dickey Fuller test performed earlier provided evidence that the data demonstrated trend stationary behavior. For this reason, a linear trend component identified as "drift" will be added. Finally, because a visual examination of the ACF plots indicates a seasonal pattern at lag 5, a seasonal term of period 5 will be added to see if it can improve the fit. Because the time series is irregular the period in this case is the ordinality of the launch in terms of launch date. Table 1 lists several that were generated.

Conclusion

There are a few conclusions about these models that are worth noting that can be seen in Appendix A. First, if we examine the coefficients for the seasonal and non-seasonal versions of ARIMA(2,0,2) we find that they are all equal to or greater than one. Since this would introduce a unit root into the predictions these models will not be considered further. The second is that all will produce fairly similar predictions. However, the seasonal versions of each model produce noticeably better results according to every measurement such as AIC, BIC, and Mean Square Error. As measured by AIC and BIC the best performing non-seasonal model, ARIMA (1,0,0) is still worst than the worst performing seasonal model, ARIMA(1,0,1)(1,0,0)[5]. The diagnostics for each of the seasonal models also show more uncorrelated and normally shaped residuals which again gives us a reason to prefer them.

Choosing a model based on diagnostic plots seems to subjective since all the plots are so similar. This leaves us with AIC, BIC, and Mean Squared Error as quantitative measures of model quality.

	ARIMA(1,0,0)(1,0,0)[5]	ARIMA(2,0,0)(1,0,0)[5]	ARIMA(1,0,1)(1,0,0)[5]
AIC	538.32	540.21	540.33
BIC	551.96	556.58	556.69
Mean Squared Error	2.64727	2.667236	2.641737

Table 1 - Comparison of Seasonal Models

As can be seen from the above table, the seasonal versions of ARIMA(1,0,0) and ARIMA(1,0,1) have the best scores. However, ARIMA(1,0,1) has a slightly better Mean Squared Error while ARIMA(1,0,0) has better AIC and BIC scores. So it may be reasonable to conclude that either one of these models could be used for making predictions.

Discussion

The previously generated models are not standard ARIMA models but instead utilize a linear trend and an ARIMA noise process. I think it would be useful to explain why these nonstandard methods were chosen. It should be acknowledged that these models have more skewed residuals and higher AIC and BIC scores than models which eliminated trend using differencing. An example of this can be shown in Figure 14 and Figure 15 for an ARIMA (1,1,0) model and an ARIMA (1,0,0) model. First, it was previously shown that the original series was already stationary, and any differencing operations would overcomplicate the model. Second, an Augmented Dickey Fuller test which incorporated a trend and a constant successfully rendered the series trend stationary. Therefore, I felt differencing the series was not justified.

Modeling the data this way also has the benefit of producing accurate predictions with narrower error boundaries. As shown below, both a differenced model and an ARIMA error model produce successful predictions. But the ARIMA error model does so with narrower boundaries. Theoretically, the more bell-shaped residuals in Figure 14 would indicate a model that incorporates more relevant information. Nevertheless, since this model produces less reliable forecasts due to the wide prediction intervals I felt it was not as useful. The reason for this difference in the residuals is not entirely clear. It may be that the differenced model "casts too wide a net" and the ARIMA error model "casts too narrow a net" since the residuals for the error model show significant skewing. This may indicate that future research could build improved models which would eliminate this skewing.

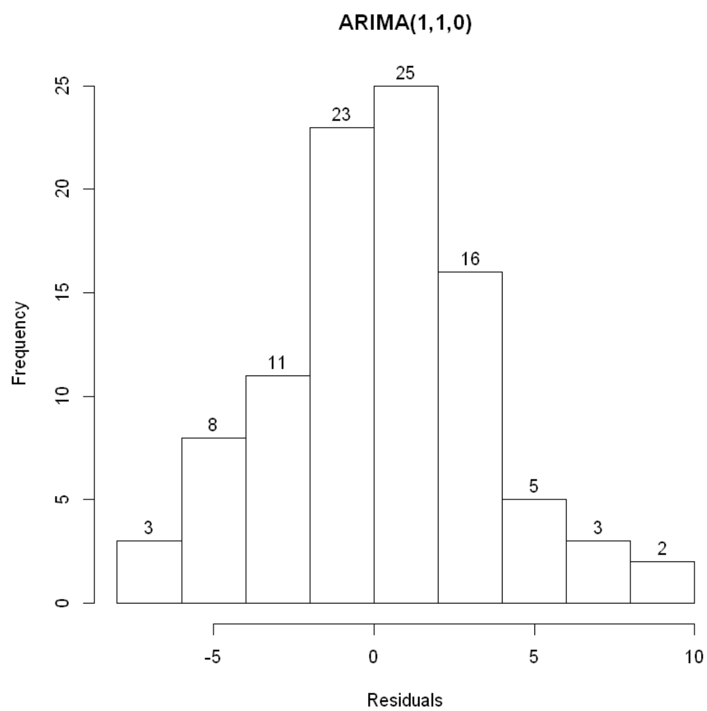

Figure 14 - Histogram of residuals for ARIMA (1,1,0)

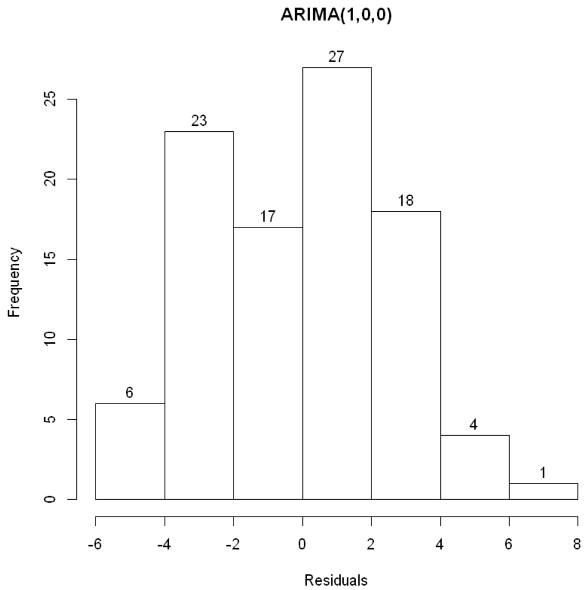

Figure 15 - Histogram of residuals for ARIMA (1,0,0)

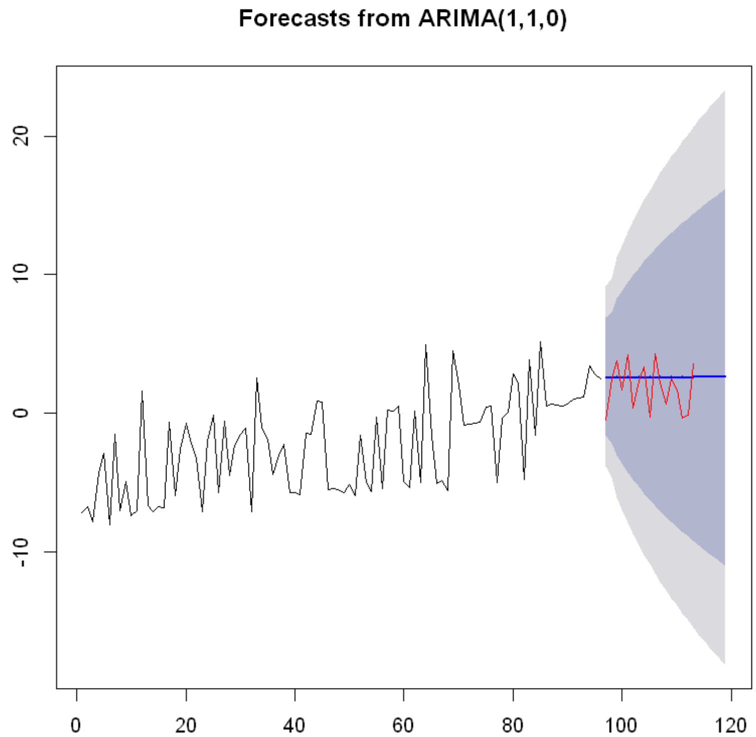

Figure 16 - Predicted vs actual for differenced model with 15% withheld for validation. The prediction interval, indicated by the blue shaded area, is much larger than that of Figure 17. This indicates that forecasts from this model are less reliable than those in Figure 17.

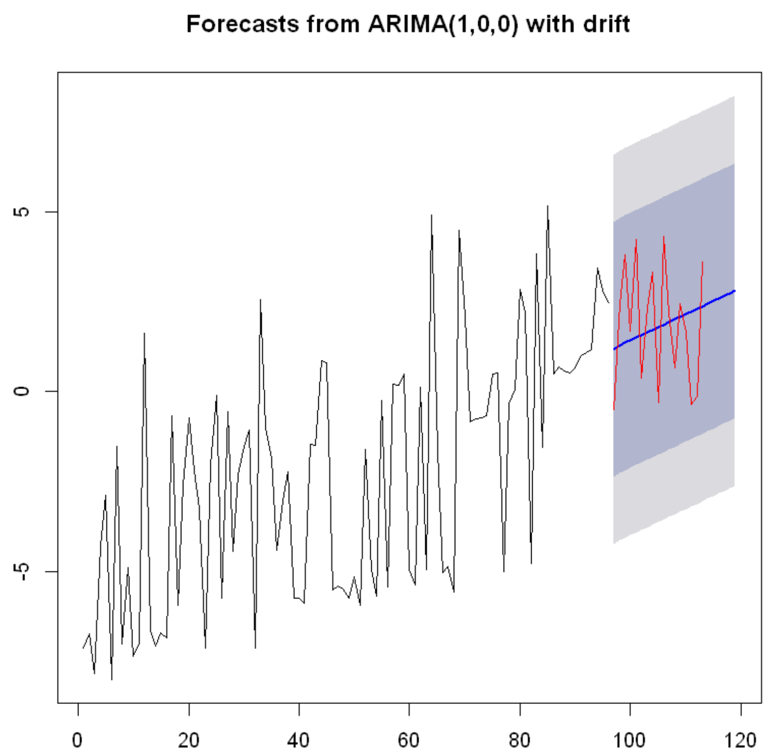

Figure 17 - Predicted vs actual for non-differenced model and 15% withheld for validation. The prediction interval, indicated by the blue shaded area, is much narrower than that of Figure 16. This indicates that forecasts from this model are more reliable than that of Figure 16.

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Appendix A: Results of Candidate Models

This appendix describes each model in detail along with predictions and diagnostics to evaluate model quality. The results contained in this appendix are summarized in the conclusion section. The models are grouped according to whether or not they include a seasonal term. It should be emphasized that the following models are ARIMA error models and take the form given in (10). Each prediction is based on ordinality and not on a particular date. As shown below, it was discovered that both the seasonal and non-seasonal versions of ARIMA(2,0,2) produced a unit root. Therefore, they are not considered further.

Model Diagnostics

Recall that the error term in an ARIMA model is independent and identically distributed. Therefore, the degree to which the residuals fit this description helps us to determine model quality. The first way of doing this is to measure the autocorrelation of the residuals since this measure's independence. The second way is by plotting a histogram of the residuals. In doing this we are seeing how well the residuals fit a hypothetical bell curve, with a more bell-shaped plot preferred. Finally, we examine the residuals with a Q-Q plot. A Q-Q plot is a visual way of checking if a data set came from a hypothetical distribution. In this case we are checking how likely the residuals came from a normal distribution. Residuals which line up more closely in a straight line are more likely to come from a normal distribution and therefore those models are to be preferred. The results from these diagnostics are considered in the conclusion.

ARIMA MODELS

	AR(1)	AR(2)	ARMA(1, 1)	ARMA(2,2)
L1.ar	-0.0410	-0.0393	-0.3392	-1.1184
L2.ar	NA	0.0441	NA	-0.8897
Intercept	-5.8386	-5.8415	-5.8386	-5.8436
L1.ma	NA	NA	0.2909	1.1131
L2.ma	NA	NA	NA	1.0000
Drift	0.0728	0.0729	0.0728	0.0730
AIC	544.11	545.89	546	543.62
BIC	555.02	559.52	559.64	562.71
	AR(1)SAR(1)	AR(2)SAR(1)	ARMA(1,1)SAR(1)	ARMA(2,2)SAR(1)
L1.ar	0.0069	0.0062	-0.6521	-1.4184
L2.ar	NA	0.0316	NA	-0.9551
Intercept	-5.8280	-5.8315	-5.8276	-5.8357
L1.ma	NA	NA	0.6523	1.4141
L2.ma	NA	NA	NA	1.0000
L1.sar	0.2615	0.2598	0.2604	0.2086
Drift	0.0724	0.0724	0.0724	0.0727
AIC	538.32	540.21	540.33	540.79
BIC	551.96	556.58	556.69	562.61

Table 2 - Candidate ARIMA Models. The terms AR(n), MA(n), and SAR(n) describe the number of Autoregressive, Moving Average, and Seasonal Autoregressive terms used. The value of each coefficient is described in the corresponding cell marked L1, L2, etc. The intercept is the estimated mean of the data while the Drift is the estimated linear trend. AIC and BIC signify Akaike's Information Criterion and Bayesian Information Criterion respectively.

Non-Seasonal Models:

Each of the following models does not include a seasonal term. As will be shown below these models perform worse on every measure as compared to the seasonal models. Nevertheless, they still perform reasonably well.

ARIMA (1,0,0)

ARIMA (1,0,0) has a moderately low AIC and BIC, although there are values which are significantly lower. This model would likely perform adequately but would not really be the first choice. The residuals are skewed negatively which indicates that this model tends to underestimate the data.

Forecast:

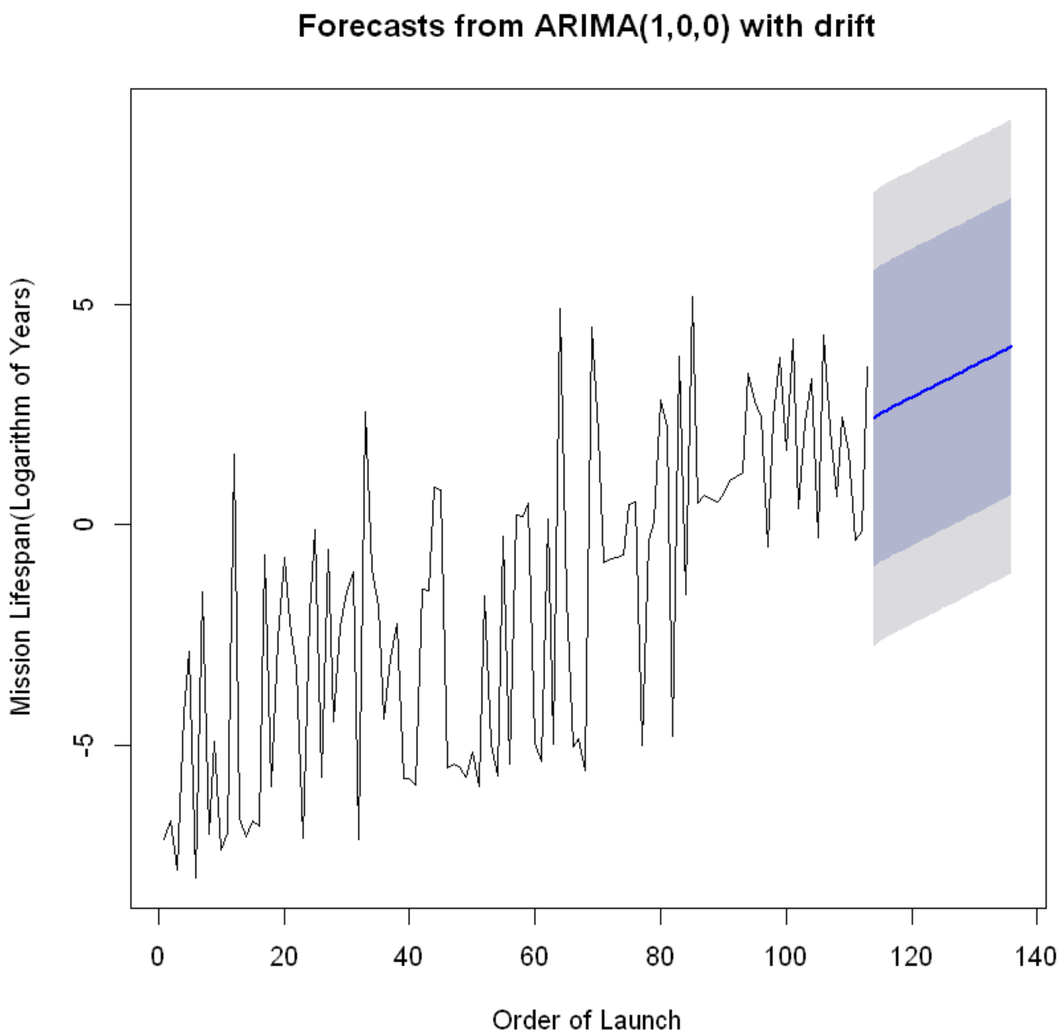

Figure 18 - Forecasts from ARIMA(1,0,0) with drift – This graph shows predictions for lifespan values of future launches. The predictions show a clear increasing trend. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The dark blue shaded area shows where 80 percent of possible future values lie. The light blue shaded area shows where 95 percent of possible values lie. A linear regression is fitted and an ARIMA model is built on the residuals. The ARIMA portion uses a lag of one value to predict future values. The formula for the 1-step ahead forecast is: Forecast = (Current Value)-0.0410 + (Ordinality * 0.0728) – 5.8386.*

23 point forecast from ARIMA(1,0,0) with drift					
	Point Forecast	Lo.80	Hi.80	Lo.95	Hi.95
114	2.414020084	-0.95539	5.783431	-2.73905	7.567089
115	2.538528578	-0.83372	5.910775	-2.61888	7.695935
116	2.609234384	-0.76302	5.981486	-2.54818	7.766648
117	2.682148196	-0.6901	6.0544	-2.47527	7.839562
118	2.754971393	-0.61728	6.127223	-2.40244	7.912385
119	2.827798309	-0.54445	6.20005	-2.32962	7.985212
120	2.900625073	-0.47163	6.272877	-2.25679	8.058039
121	2.973451843	-0.3988	6.345703	-2.18396	8.130866
122	3.046278613	-0.32597	6.41853	-2.11114	8.203692
123	3.119105382	-0.25315	6.491357	-2.03831	8.276519
124	3.191932152	-0.18032	6.564184	-1.96548	8.349346
125	3.264758921	-0.10749	6.637011	-1.89265	8.422173
126	3.337585691	-0.03467	6.709837	-1.81983	8.494999
127	3.410412461	0.038161	6.782664	-1.747	8.567826
128	3.48323923	0.110988	6.855491	-1.67417	8.640653
129	3.556066	0.183814	6.928318	-1.60135	8.71348
130	3.62889277	0.256641	7.001144	-1.52852	8.786307
131	3.701719539	0.329468	7.073971	-1.45569	8.859133
132	3.774546309	0.402295	7.146798	-1.38287	8.93196
133	3.847373079	0.475121	7.219625	-1.31004	9.004787
134	3.920199848	0.547948	7.292451	-1.23721	9.077614
135	3.993026618	0.620775	7.365278	-1.16439	9.15044
136	4.065853388	0.693602	7.438105	-1.09156	9.223267

Table 3 - Predictions from ARIMA(1,0,0) with drift – This table contains the detailed predictions plotted in Figure 18 along with their 80 percent and 95 percent prediction intervals that were shown in the dark blue and light blue areas respectively.

Predicted vs Actual:

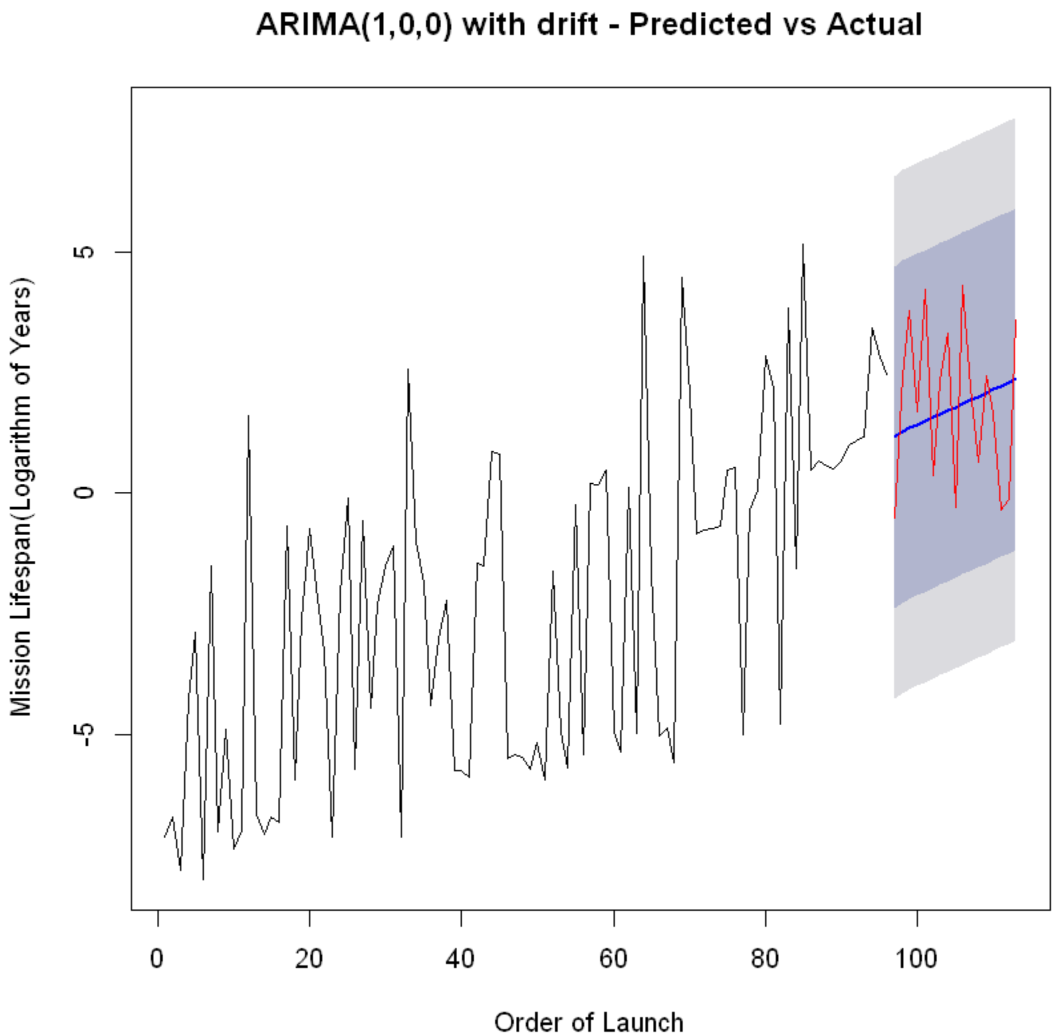

Figure 19 - ARIMA(1,0,0) with drift - Predicted vs Actual – 15 percent of the original data is withheld and tested against a model generated with the remaining 85 percent. All withheld values fall within the dark blue area indicating that the model predicts the actual values reasonably well. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The withheld data is shown in red and the forecast generated with the model is shown in blue. 80 percent of projected values fall within the dark blue shaded area and 95 percent of all projected values fall within the light blue shaded area.

Diagnostics:

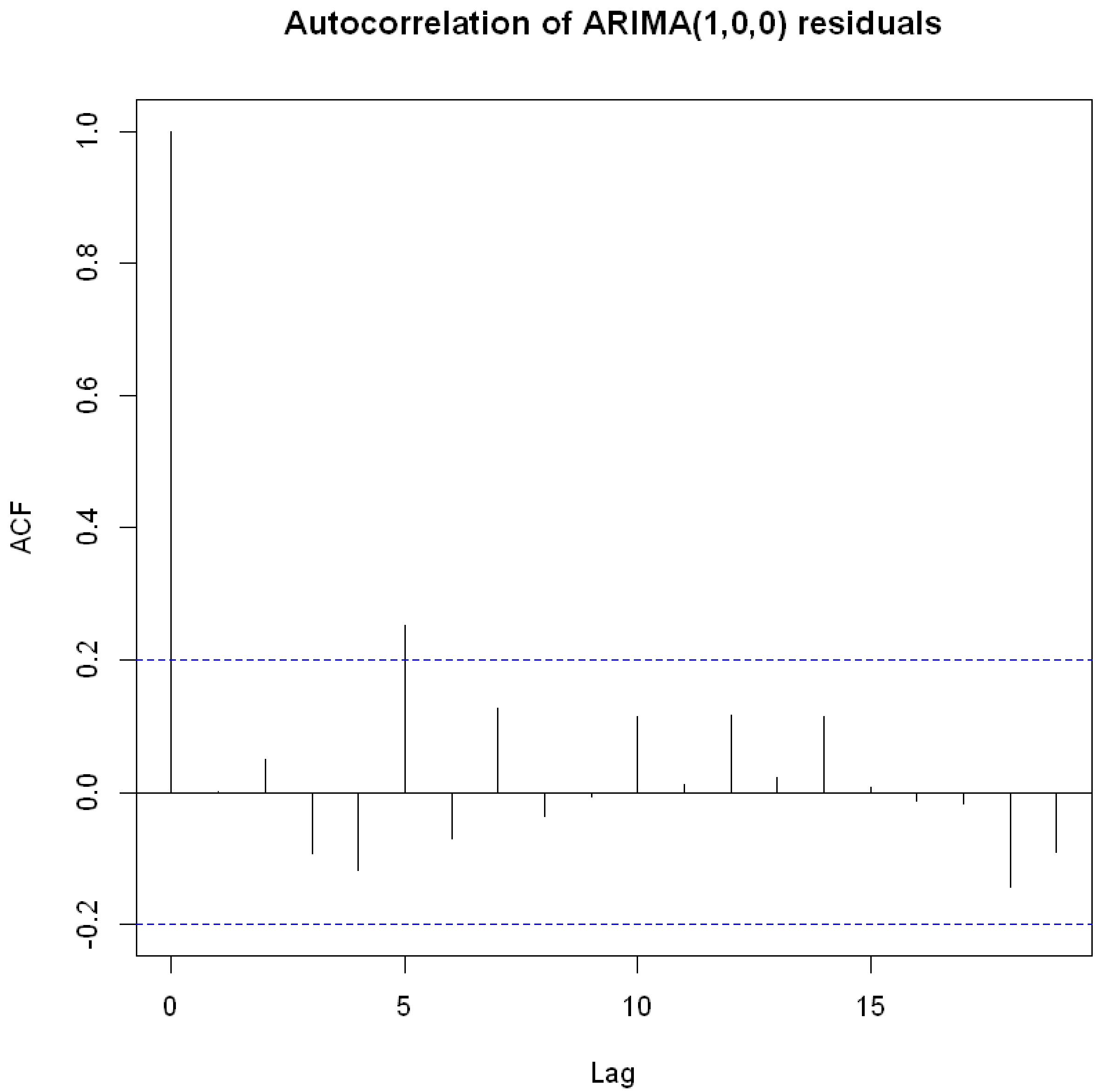

Figure 20 - Autocorrelation of ARIMA(1,0,0) residuals – Residuals are the difference between the predictions of the model and the actual value. Here is a plot of the correlation of each residual between itself and those of previous lags. Every correlation beyond 0.2 signifies statistical significance. The spike above the dotted line at lag 5 shows significant correlation every 5 lags which indicates possible seasonality.

ARIMA(1,0,0)			
Predicted	Actual	Error	Squared Error
1.164189414	-0.49989	1.664079	2.76916
1.282807079	2.44369	1.160883	1.34765
1.353962	3.799628	2.445666	5.981281
1.426707387	1.683646	0.256939	0.066017
1.499399478	4.227464	2.728064	7.442335
1.572093355	0.369321	1.202772	1.446662
1.644787173	2.330355	0.685568	0.470004
1.717480992	3.319869	1.602388	2.567649
1.790174811	-0.29803	2.088209	4.360615
1.86286863	4.315931	2.453062	6.017514
1.93556245	2.188834	0.253272	0.064147
2.008256269	0.647592	1.360664	1.851407
2.080950088	2.441586	0.360636	0.130058
2.153643907	1.655872	0.497771	0.247776
2.226337726	-0.3545	2.580841	6.660741
2.299031546	-0.12559	2.424626	5.87881
2.371725365	3.600527	1.228801	1.509953
Mean Squared Error			2.871281

Table 4 - ARIMA(1,0,0) Mean Square Error – This table shows how prediction generated by the model in Figure 19 is compared to the actual value. Each error is then squared, and the arithmetic mean of each of the squared errors is computed. The result shows an average squared error of 2.871281. The result shows that this model was significantly worse at generating predictions than the version that assumes seasonality.

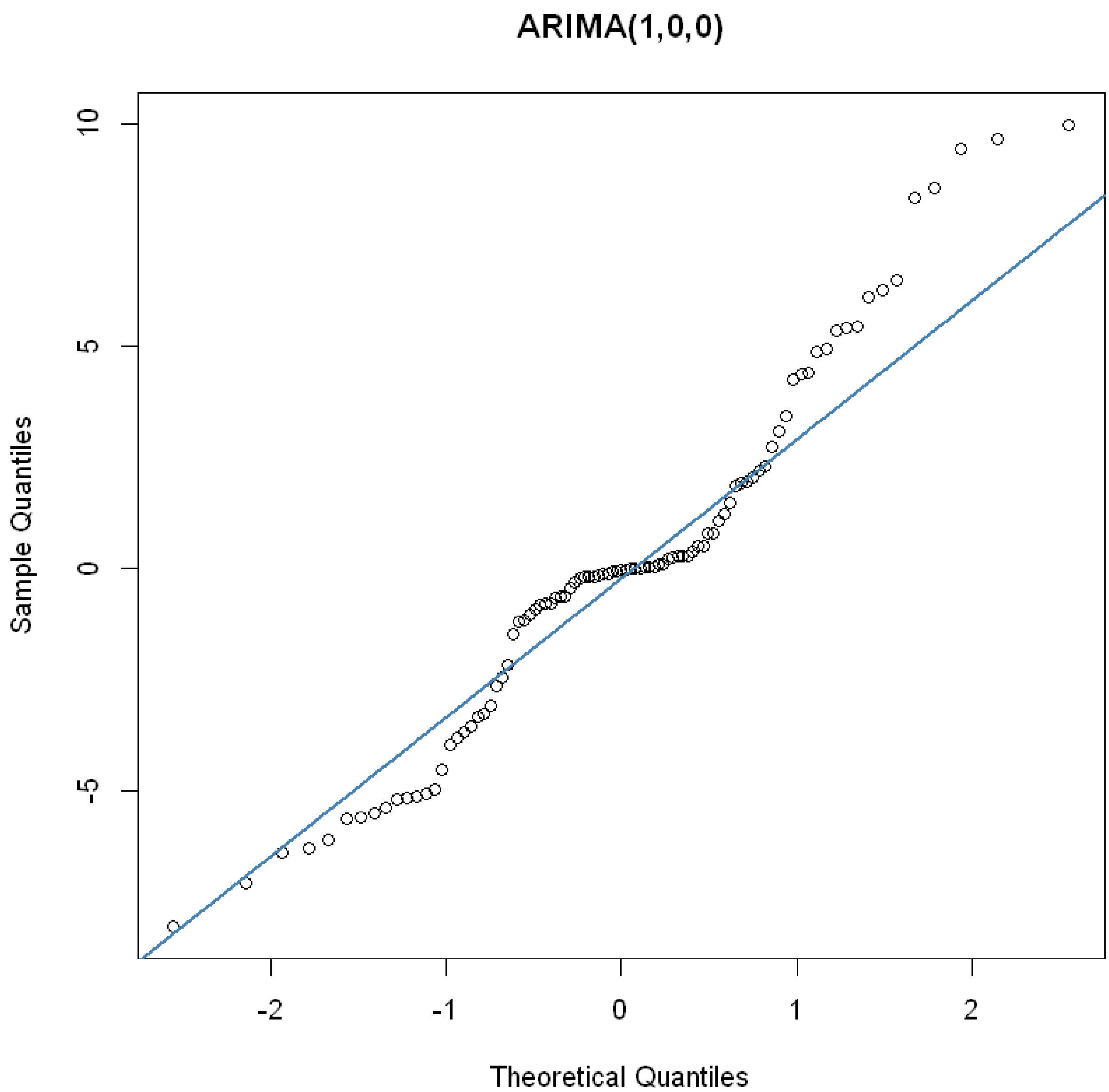

Figure 21 - QQ Plot for ARIMA(1,0,0) – Residuals in an ARIMA model are theoretically normally distributed. The more diagonal the points are on a QQ plot, the more normally distributed they are. This plot shows that the residuals are not normally distributed along certain points where they do not line up very well with the blue line. This shows that some information is not being captured by the model.

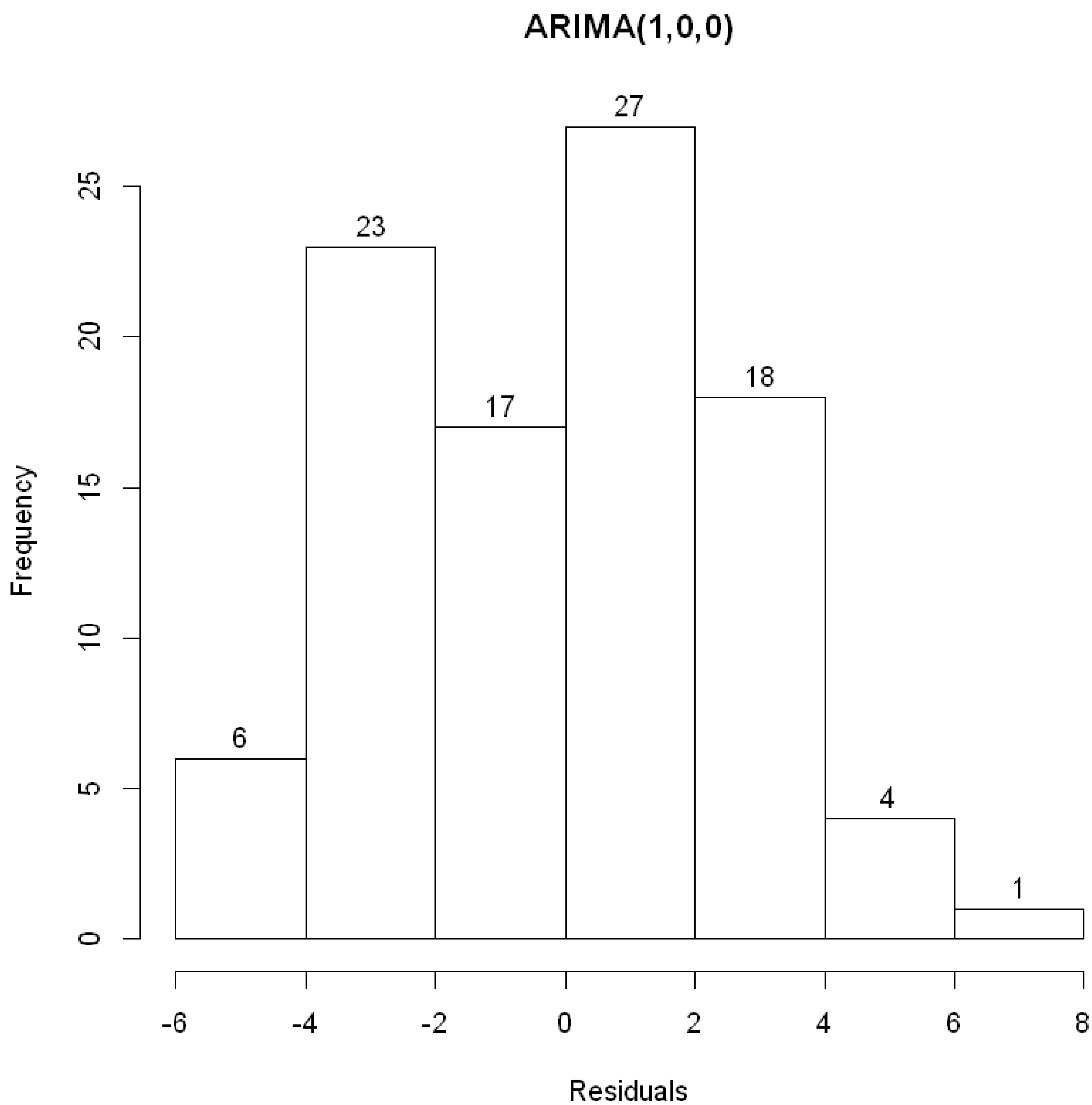

Figure 22 - Histogram of Residuals for ARIMA(1,0,0) – Another way to measure normality of residuals is by examining a histogram of the residuals to see how much it resembles a bell curve. Again, we see skewing in the -2 to -4 bin. Just like Figure 21, this shows that some information is not being captured by the model.

ARIMA (2,0,0)

ARIMA(2,0,0) has a higher AIC,BIC, and Mean Square Error than ARIMA(1,0,0). However, these differences aren’t significant so both of these models should be equally effective.

Forecast:

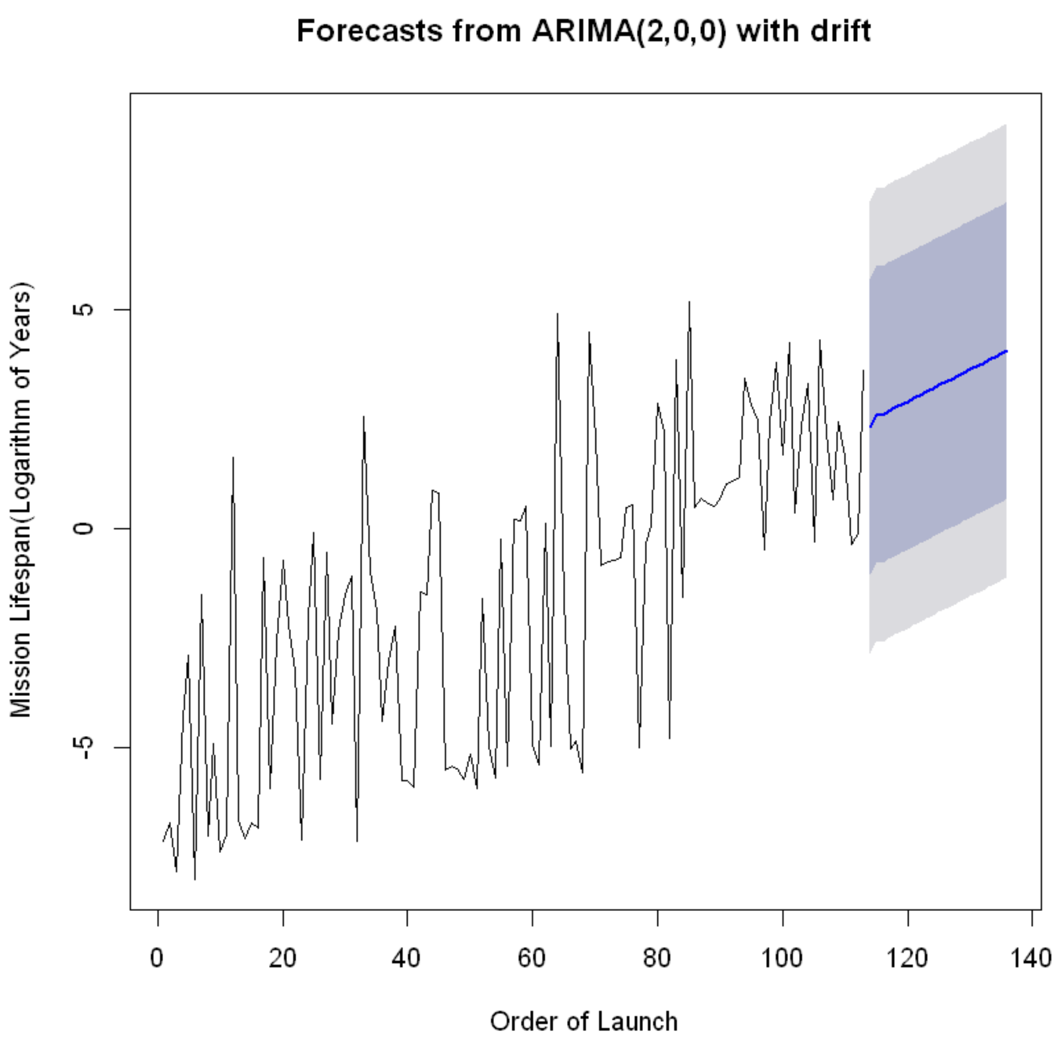

*Figure 23 - Forecasts from ARIMA(2,0,0) with drift – This graph shows predictions for lifespan values of future launches. The predictions show a clear increasing trend. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The dark blue shaded area shows where 80 percent of possible future values lie. The light blue shaded area shows where 95 percent of possible values lie. A linear regression is fitted and an ARIMA model is built using the residuals. The ARIMA portion of the model uses two previous lags to predict future values. The formula for each 1-step ahead forecast is: Forecast = (Current Value) * -0.0393 + (Lag 1 Value) * 0.0441 + (Ordinality * 0.0729) – 5.8415.*

23 point forecast from ARIMA (2,0,0) with drift					
	Point Forecast	Lo.80	Hi.80	Lo.95	Hi.95
114	2.308364985	-1.0731	5.689829	-2.86314	7.479868
115	2.59602078	-0.78805	5.980092	-2.57947	7.771512
116	2.600210208	-0.78738	5.987803	-2.58067	7.781086
117	2.685237011	-0.70238	6.07285	-2.49567	7.866145
118	2.754581737	-0.63304	6.142203	-2.42634	7.935501
119	2.828109066	-0.55951	6.21573	-2.35281	8.009029
120	2.900780196	-0.48684	6.288402	-2.28014	8.0817
121	2.973669497	-0.41395	6.361291	-2.20725	8.154589
122	3.046512451	-0.34111	6.434134	-2.13441	8.227432
123	3.119366853	-0.26825	6.506988	-2.06155	8.300287
124	3.19221876	-0.1954	6.57984	-1.9887	8.373139
125	3.265071269	-0.12255	6.652693	-1.91585	8.445991
126	3.337923645	-0.0497	6.725545	-1.843	8.518843
127	3.410776053	0.023155	6.798397	-1.77014	8.591696
128	3.483628454	0.096007	6.87125	-1.69729	8.664548
129	3.556480857	0.168859	6.944102	-1.62444	8.737401
130	3.629333259	0.241712	7.016955	-1.55159	8.810253
131	3.702185661	0.314564	7.089807	-1.47873	8.883105
132	3.775038063	0.387417	7.162659	-1.40588	8.955958
133	3.847890465	0.460269	7.235512	-1.33303	9.02881
134	3.920742867	0.533121	7.308364	-1.26018	9.101663
135	3.99359527	0.605974	7.381217	-1.18732	9.174515
136	4.066447672	0.678826	7.454069	-1.11447	9.247368

Table 5 - Predictions from ARIMA(2,0,0) with drift - This table contains the detailed predictions plotted in Figure 23 along with their 80 percent and 95 percent prediction intervals that were shown in the dark blue and light blue areas respectively.

Predicted vs Actual:

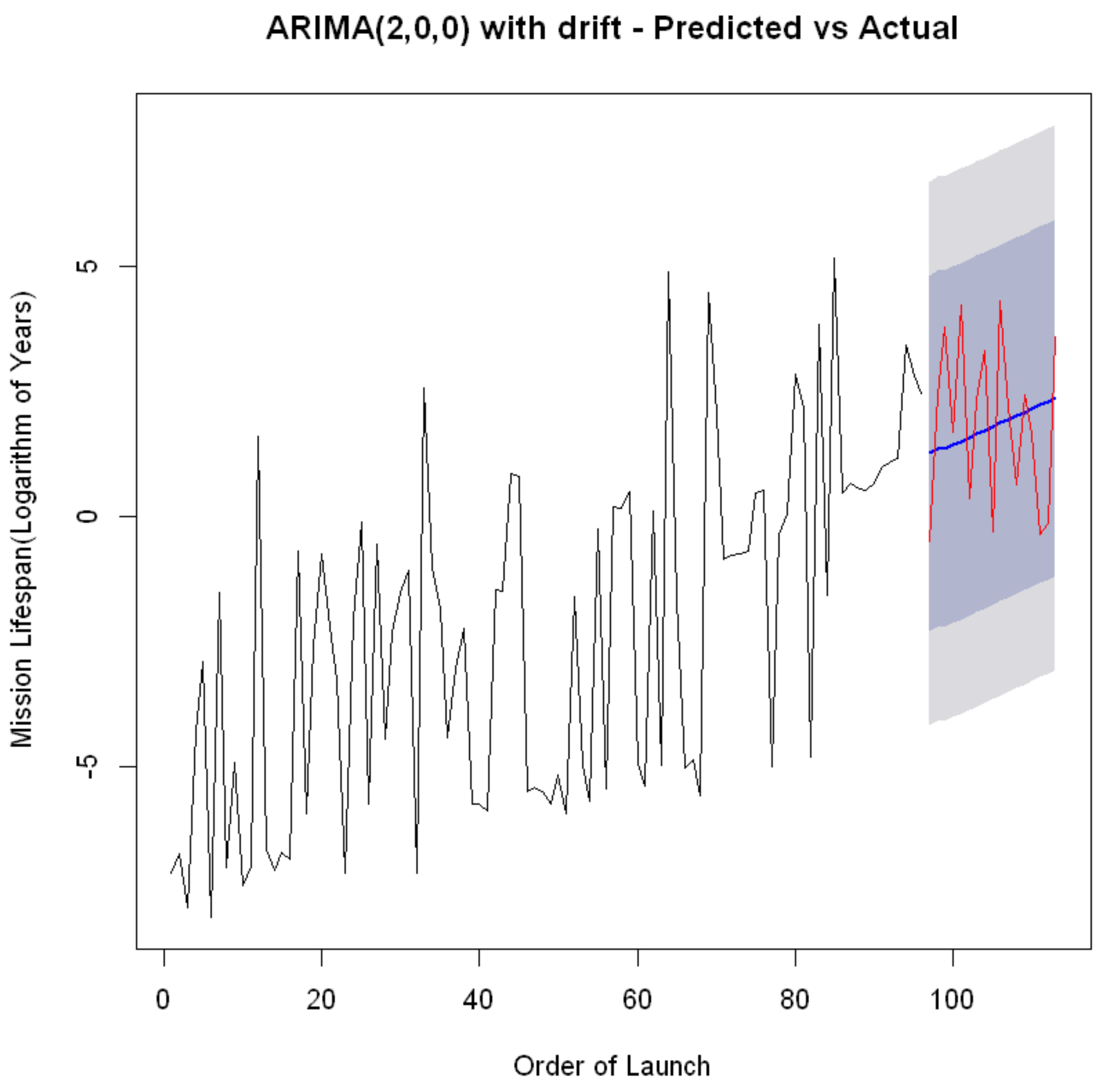

Figure 24 - ARIMA(2,0,0) with drift - Predicted vs Actual – 15 percent of the original data is withheld and tested against a model generated with the remaining 85 percent. All withheld values fall within the dark blue area indicating that the model predicts the actual values reasonably well. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The withheld data is shown in red and the forecast generated with the model is shown in blue. 80 percent of projected values fall within the dark blue shaded are and 95 percent of all projected values fall within the light blue shaded area.

Diagnostics:

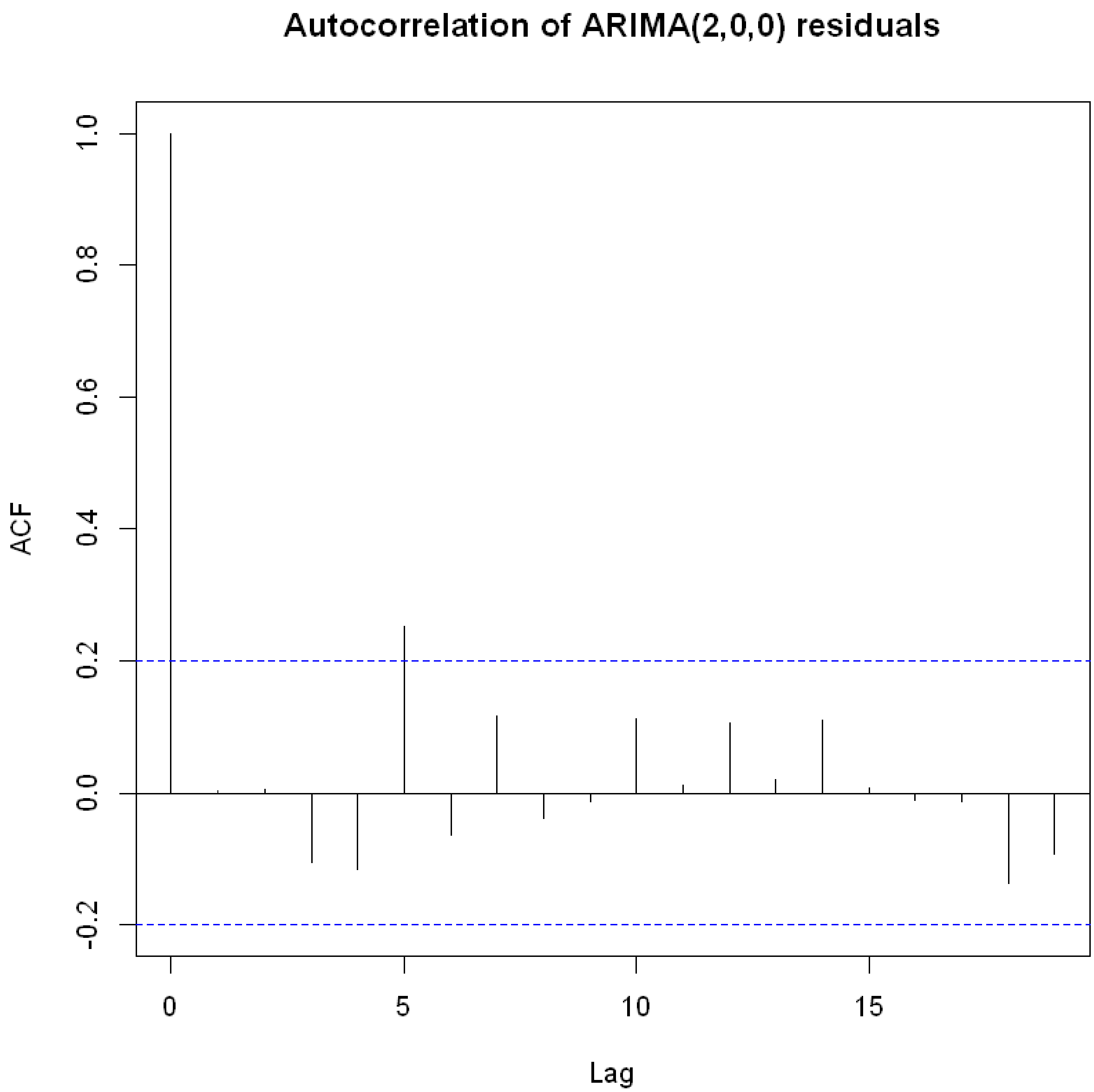

Figure 25 - Autocorrelation of ARIMA(2,0,0) residuals - Here is a plot of the correlation of each residual between itself and those of previous lags as in Figure 20. Again, there is a spike at lag 5 indicating possible seasonality and showing that some information is being missed by the model.

ARIMA(2,0,0)			
Predicted	Actual	Error	Squared Error
1.265720931	-0.49989	1.765611	3.117381
1.35663382	2.44369	1.087056	1.181691
1.364302771	3.799628	2.435325	5.930807
1.440156674	1.683646	0.243489	0.059287
1.509624538	4.227464	2.717839	7.38665
1.5827666	0.369321	1.213446	1.472451
1.655467743	2.330355	0.674888	0.455473
1.728369919	3.319869	1.591499	2.532871
1.801243309	-0.29803	2.099277	4.406965
1.874127846	4.315931	2.441803	5.962402
1.947010565	2.188834	0.241824	0.058479
2.01989391	0.647592	1.372302	1.883213
2.092777142	2.441586	0.348809	0.121668
2.165660409	1.655872	0.509788	0.259884
2.23854367	-0.3545	2.593047	6.723893
2.311426932	-0.12559	2.437021	5.939072
2.384310195	3.600527	1.216216	1.479182
Mean Squared Error			2.880669

Table 6 - ARIMA(2,0,0) Mean Square Error - Each prediction generated by the model in Figure 24 is compared to the actual value. Each error is then squared, and the arithmetic mean of each of the squared errors is computed. The result shows an average squared error of 2.8880669. The result shows that this model was significantly worse at generating predictions than the version that assumes seasonality.

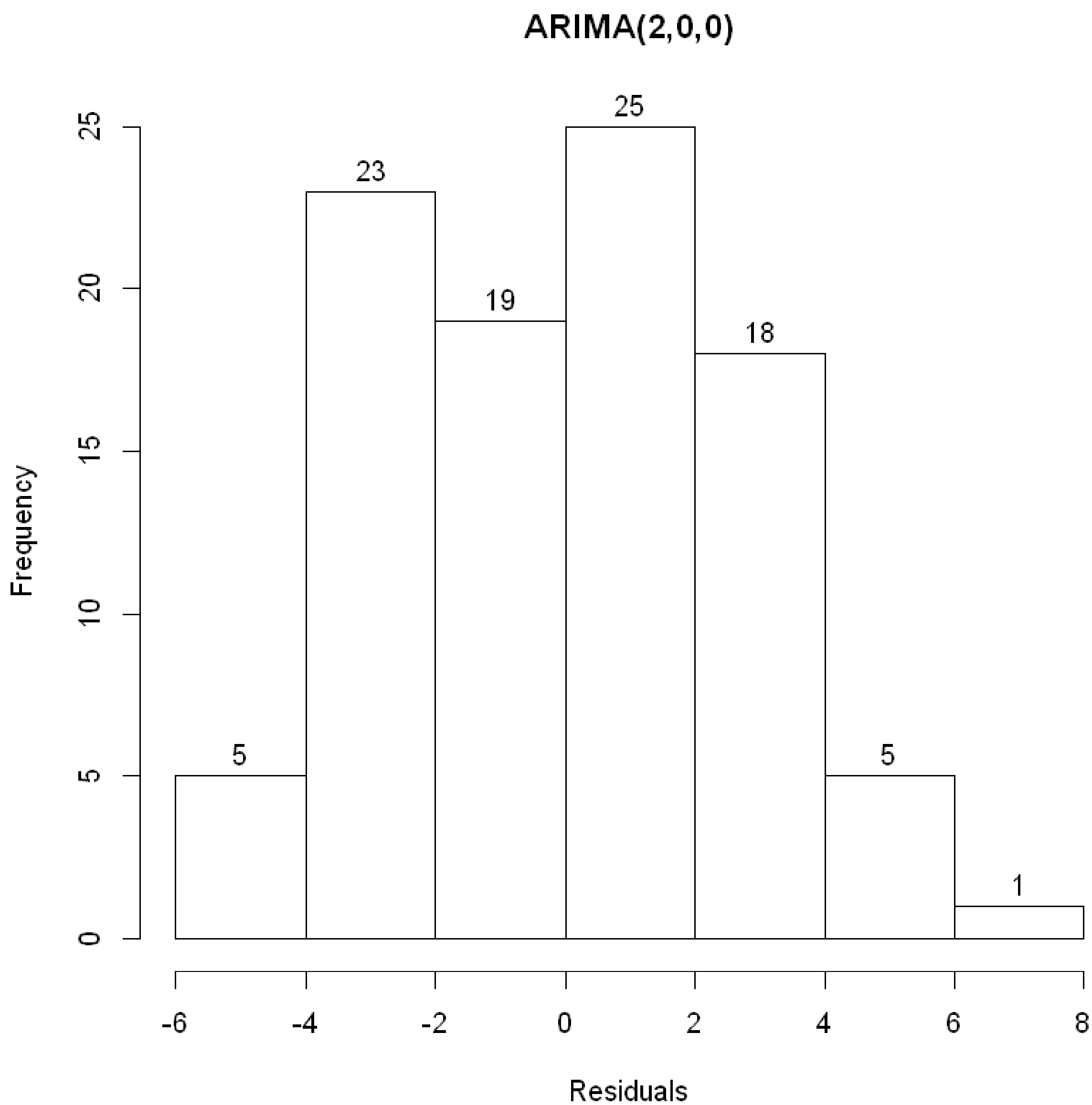

Figure 26 - Histogram of Residuals for ARIMA(2,0,0) – The normality of the model's residuals is measured as in Figure 22. The two histogram's are extremely similar and show that some information is not being captured by this model.

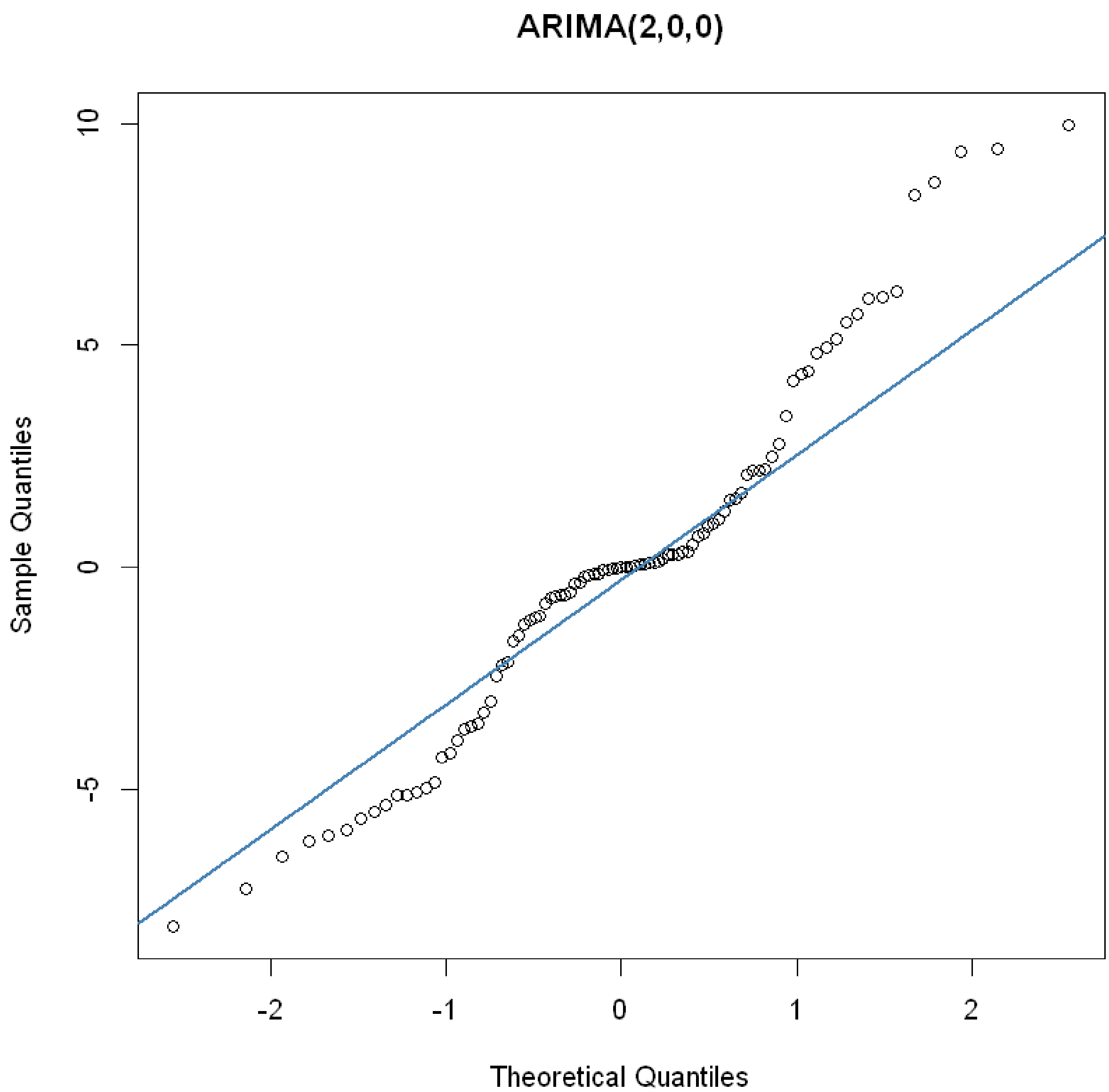

Figure 27 - QQ Plot for ARIMA(2,0,0) – Residuals in an ARIMA model are theoretically normally distributed. The more diagonal the points are on a QQ plot, the more normally distributed they are. This plot shows that the residuals are not normally distributed along certain points where they do not line up very well with the blue line. This shows that some information is not being captured by the model.

ARIMA (1,0,1)

ARIMA(1,0,1) has the highest AIC and BIC of any of the non-seasonal models. While it would be adequate it would not be an ideal choice. In addition, the residuals are the most negatively skewed which further reduces its effectiveness.

Forecast:

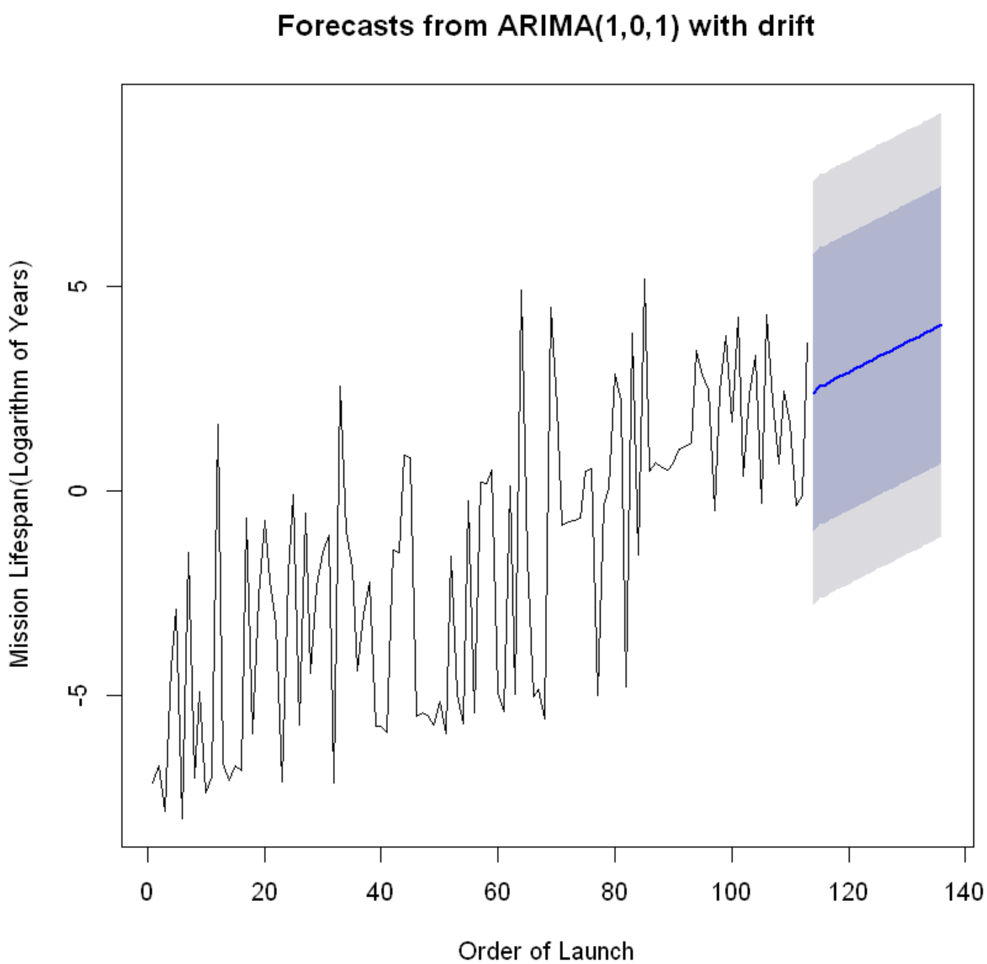

*Figure 28 - Forecasts from ARIMA(1,0,1) with drift – This graph shows predictions for lifespan values of future launches. The predictions show a clear increasing trend. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The dark blue shaded area shows where 80 percent of possible future values lie. The light blue shaded area shows where 95 percent of possible values lie. A linear regression is fitted and an ARIMA model is applied to the residuals. The ARIMA portion of the model uses one previous lag and one previous residual to predict future values. The 1-step ahead forecast is: Forecast = (Current Value) * -0.3392 + (Current Residual) * 0.2909 + (Ordinality * 0.0728) – 5.8386.*

23 point forecast from ARIMA(1,0,1) with drift					
	Point Forecast	Lo.80	Hi.80	Lo.95	Hi.95
114	2.380243	-1.00293	5.763414	-2.79387	7.554357
115	2.564318	-0.8228	5.95144	-2.61584	7.744475
116	2.599402	-0.78817	5.986979	-2.58145	7.780253
117	2.685029	-0.7026	6.072658	-2.4959	7.86596
118	2.75351	-0.63412	6.141145	-2.42743	7.934451
119	2.827808	-0.55983	6.215443	-2.35313	8.008749
120	2.900132	-0.4875	6.287768	-2.28081	8.081074
121	2.973126	-0.41451	6.360761	-2.20782	8.154067
122	3.045892	-0.34174	6.433528	-2.13505	8.226834
123	3.118736	-0.2689	6.506372	-2.06221	8.299678
124	3.191554	-0.19608	6.579189	-1.98939	8.372495
125	3.26438	-0.12326	6.652016	-1.91656	8.445322
126	3.337203	-0.05043	6.724839	-1.84374	8.518145
127	3.410028	0.022392	6.797664	-1.77091	8.59097
128	3.482852	0.095216	6.870488	-1.69809	8.663794
129	3.555676	0.16804	6.943312	-1.62527	8.736618
130	3.6285	0.240865	7.016136	-1.55244	8.809442
131	3.701324	0.313689	7.08896	-1.47962	8.882266
132	3.774149	0.386513	7.161784	-1.40679	8.95509
133	3.846973	0.459337	7.234609	-1.33397	9.027915
134	3.919797	0.532161	7.307433	-1.26114	9.100739
135	3.992621	0.604985	7.380257	-1.18832	9.173563
136	4.065445	0.67781	7.453081	-1.1155	9.246387

Table 7 - Predictions from ARIMA(1,0,1) with drift - This table contains the detailed predictions plotted in Figure 28 along with their 80 percent and 95 percent prediction intervals that were shown in the dark blue and light blue areas respectively.

Predicted vs Actual:

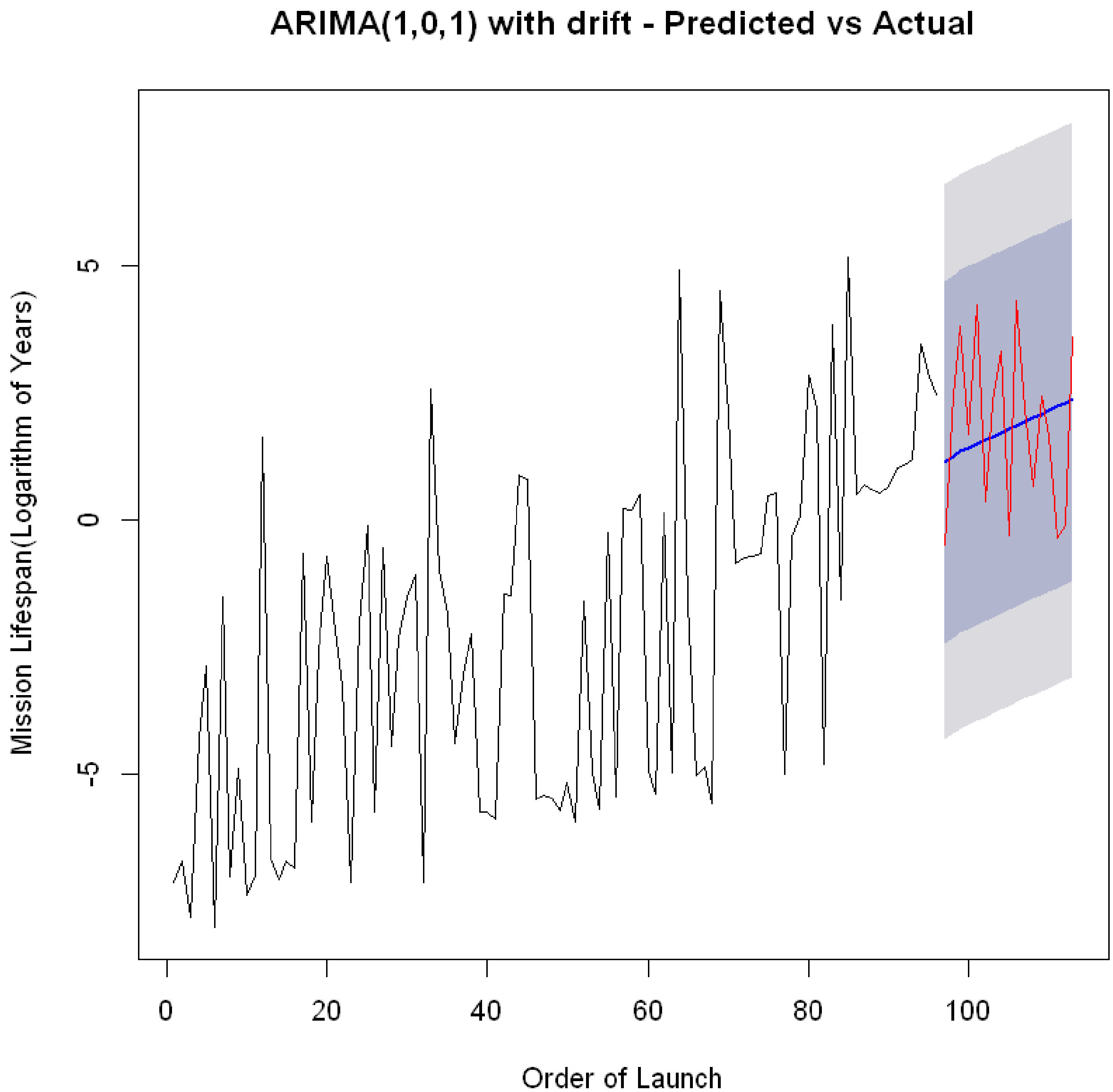

Figure 29 - ARIMA(1,0,1) with drift - Predicted vs Actual - 15 percent of the original data is withheld and tested against a model generated with the remaining 85 percent. All withheld values fall within the dark blue area indicating that the model predicts the actual values reasonably well. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The withheld data is shown in red and the forecast generated with the model is shown in blue. 80 percent of projected values fall within the dark blue shaded are and 95 percent of all projected values fall within the light blue shaded area.

Diagnostics:

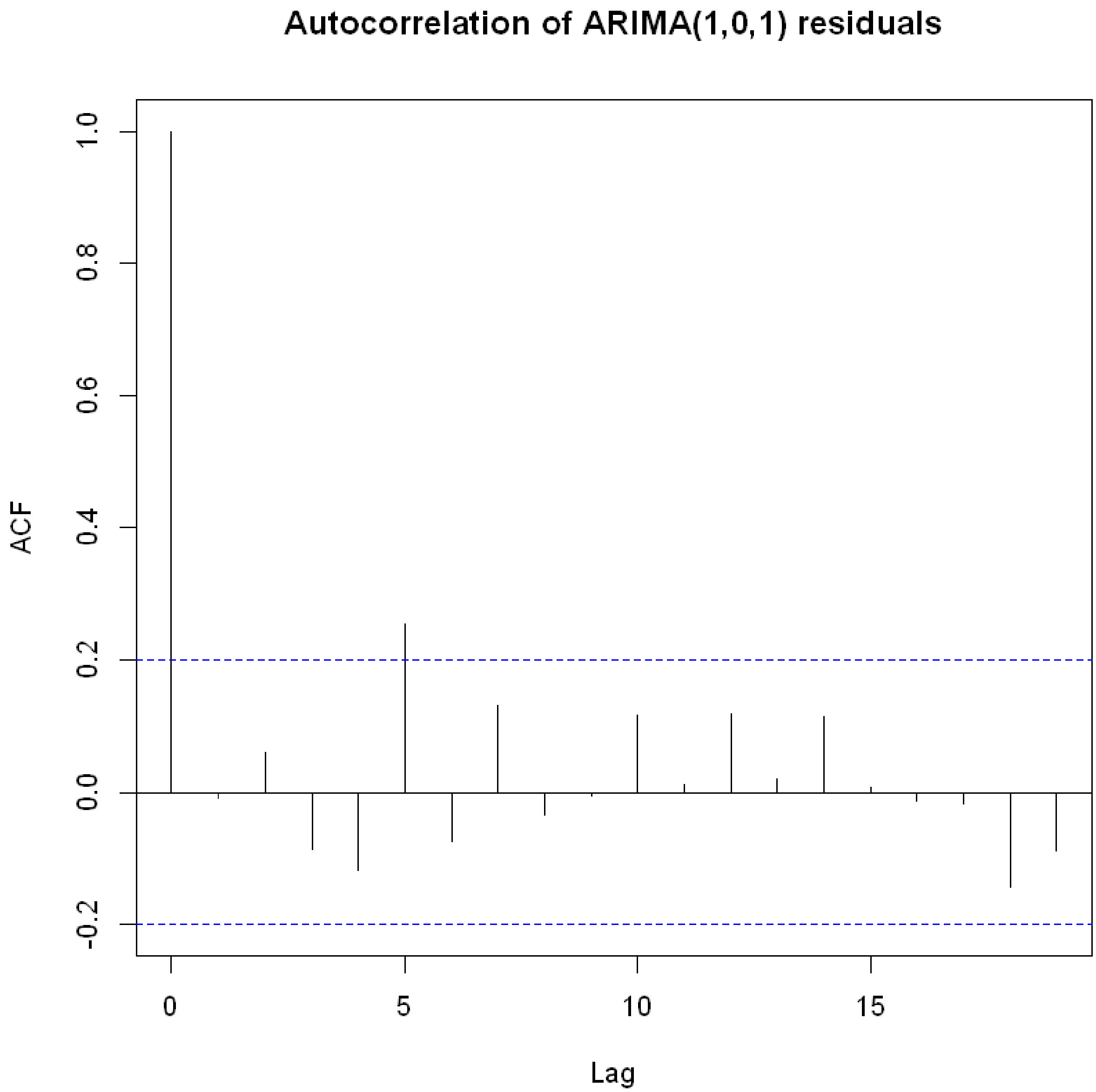

Figure 30 - Autocorrelation of ARIMA(1,0,1) residuals – This autocorrelation plot has no significant differences from those in Figure 20 and Figure 25. Again, there is a significant correlation at lag 5 which indicates that a seasonal term may improve the model.

ARIMA(1,0,1)			
Predicted	Actual	Error	Squared Error
1.141511981	-0.49989	1.641402	2.6942
1.251842559	2.44369	1.191848	1.420501
1.339777922	3.799628	2.45985	6.050861
1.418619241	1.683646	0.265027	0.070239
1.493767734	4.227464	2.733696	7.473094
1.567416678	0.369321	1.198096	1.435434
1.640456699	2.330355	0.689899	0.47596
1.713249455	3.319869	1.60662	2.581228
1.785941802	-0.29803	2.083976	4.342955
1.858593377	4.315931	2.457337	6.038507
1.931228396	2.188834	0.257606	0.066361
2.003856691	0.647592	1.356265	1.839454
2.076482256	2.441586	0.365104	0.133301
2.149106713	1.655872	0.493234	0.24328
2.221730719	-0.3545	2.576234	6.636982
2.294354543	-0.12559	2.419949	5.856152
2.366978293	3.600527	1.233548	1.521641
Mean Squared Error			2.875303

Table 8 - ARIMA(1,0,1) Mean Square Error - Each prediction generated by the model in Figure 29 is compared to the actual value. Each error is then squared, and the arithmetic mean of each of the squared errors is computed. The result shows an average squared error of 2.875303. The result shows that this model was significantly worse at generating predictions than the version that assumes seasonality.

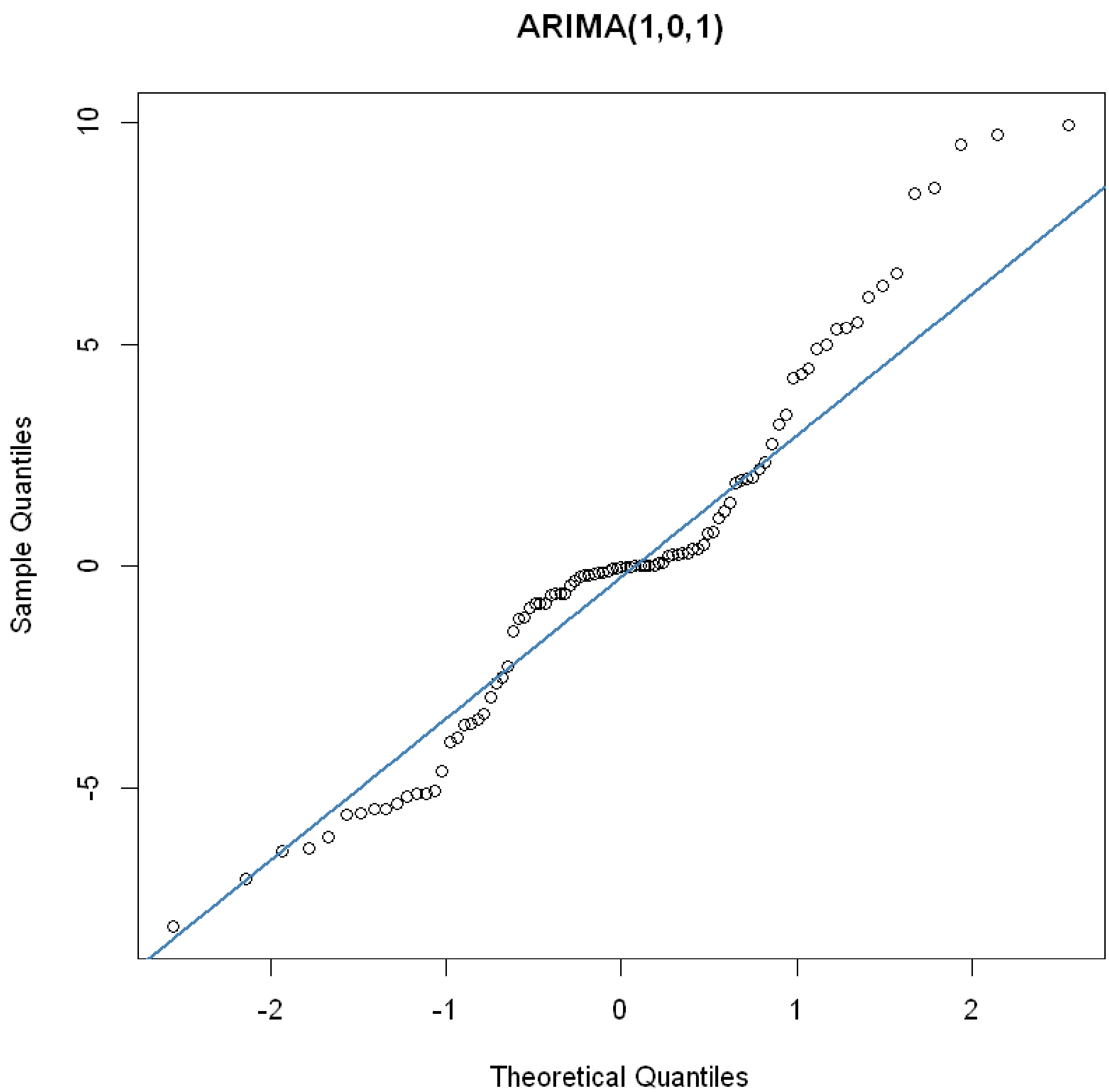

Figure 31 - QQ Plot for ARIMA(1,0,1) – Visually this QQ Plot displays the least normality of any of those generated. This plot shows that the residuals are not normally distributed along certain points where they do not line up very well with the blue line. This shows that some information is not being captured by the model.

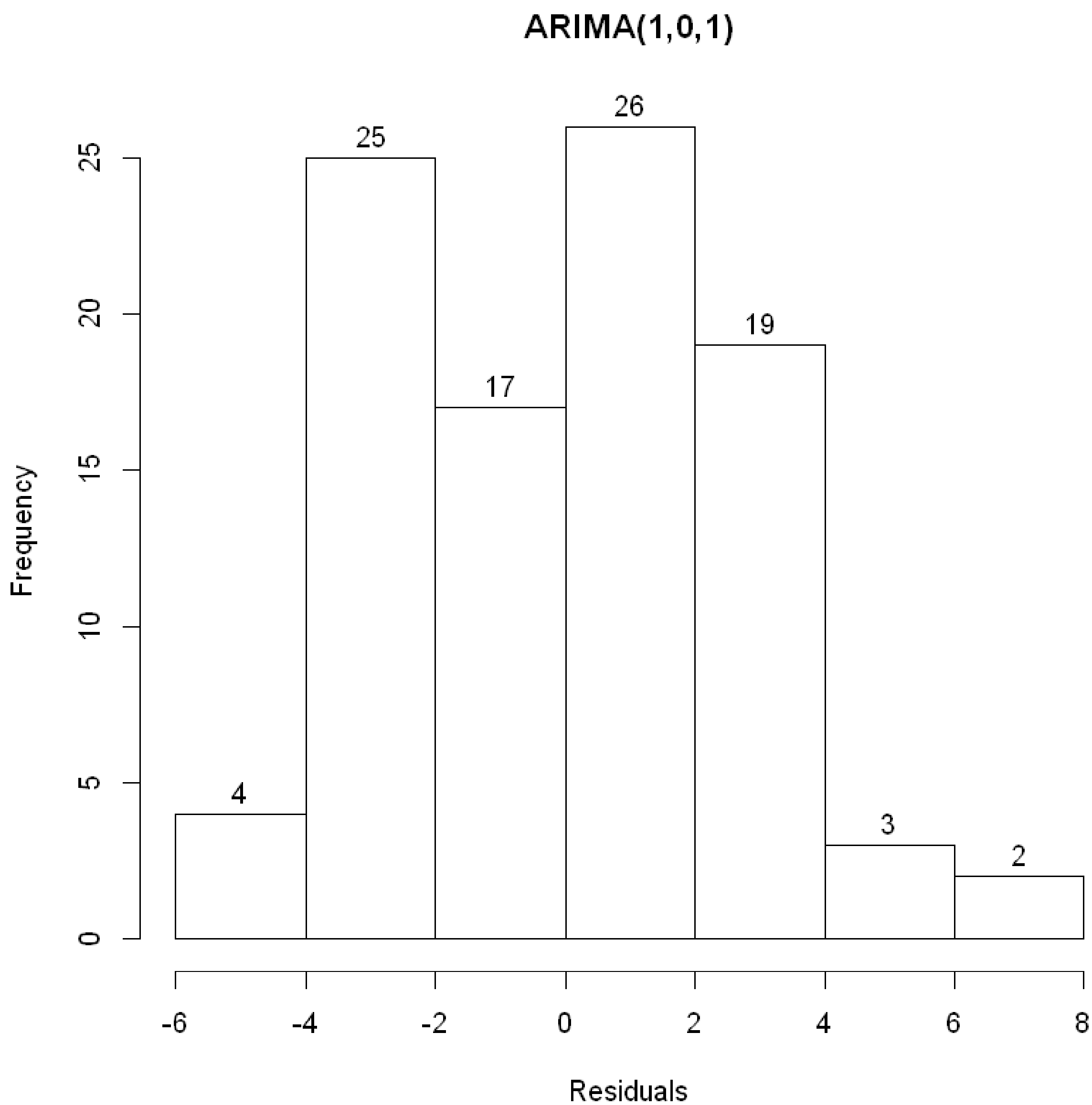

Figure 32 - Histogram of Residuals for ARIMA(1,0,1) – The histogram of the residuals shows the strongest skewing of any model generated, especially in the -2 to -4 bin with a frequency of 25. This indicates that it is missing the most significant information of any model and thus is the worst of any model generated.

Seasonal Models

In general, the seasonal models perform better than the non-seasonal models by every measure. This provides strong evidence that there is a real seasonal component to this data, possibly due to NASA waiting for favorable astronomical conditions for longer missions. A visual examination of the Predicted vs Actual graphs show that seasonal models do a better job of predicting positive or negative logarithmic values of lifespan.

ARIMA (1,0,0)(1,0,0)[5]

ARIMA(1,0,0) with a 5 period seasonal term has the lowest AIC and BIC scores of any of the models. So, in terms of parsimony, this model is the ideal choice.

Forecast:

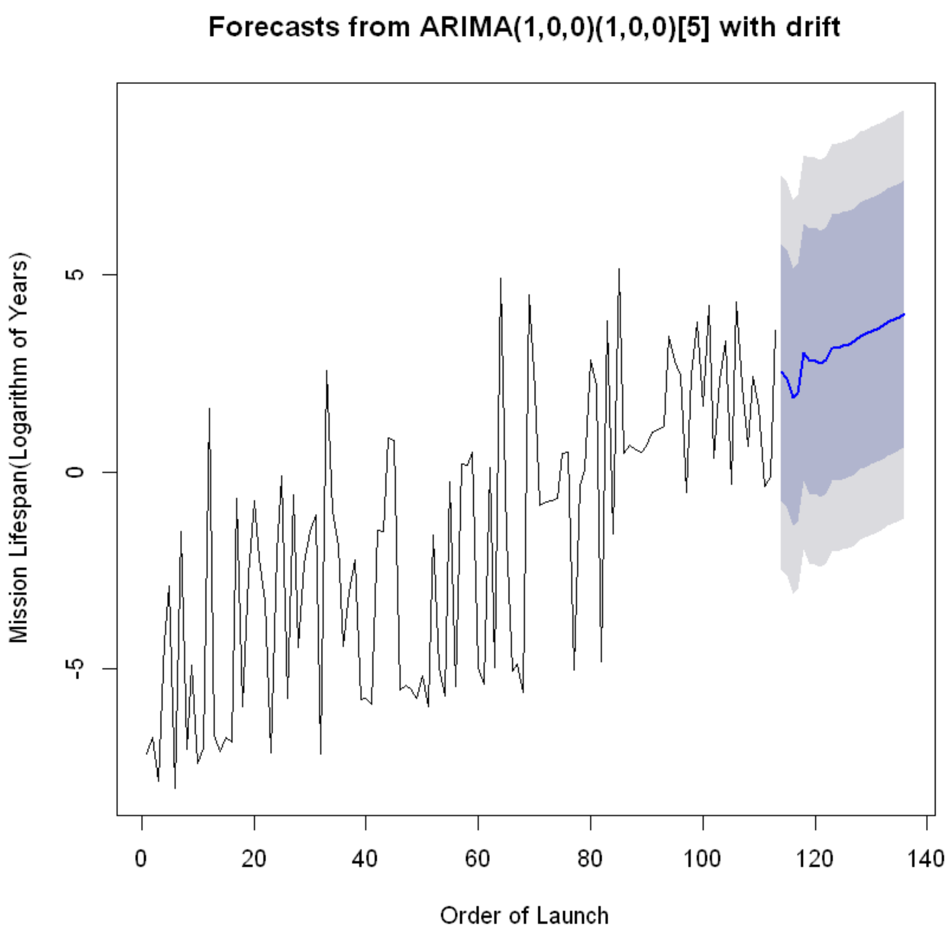

*Figure 33 - Forecasts from ARIMA(1,0,0)(1,0,0)[5] with drift - This graph shows predictions for lifespan values of future launches. The predictions show a clear increasing trend. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The dark blue shaded area shows where 80 percent of possible future values lie. The light blue shaded area shows where 95 percent of possible values lie. A linear regression is fitted and an ARIMA model is applied to the residuals. The non-seasonal portion is Current Value = (Lag 1 Value) * 0.0069. The seasonal portion is $Current\ Value_5$ = (Lag_5) * 0.2615, the subscript signifying the seasonal period. The 1-step ahead forecast is: Forecast = (Non – Seasonal Forecast) * (Seasonal Forecast) + (Ordinality * 0.0724) – 5.8280.*

23 point forecast from ARIMA(1,0,0)(1,0,0)[5] with drift					
	Point Forecast	Lo.80	Hi.80	Lo.95	Hi.95
114	2.534172	-0.73097	5.79931	-2.45943	7.527769
115	2.371187	-0.89403	5.636402	-2.62253	7.364903
116	1.898794	-1.36642	5.164009	-3.09492	6.89251
117	2.012113	-1.2531	5.277329	-2.9816	7.005829
118	3.040056	-0.22516	6.305271	-1.95366	8.033772
119	2.814627	-0.5604	6.189657	-2.34704	7.976289
120	2.825455	-0.54958	6.20049	-2.33621	7.987126
121	2.755365	-0.61967	6.1304	-2.40631	7.917035
122	2.838455	-0.53658	6.21349	-2.32322	8.000125
123	3.160746	-0.21429	6.53578	-2.00092	8.322416
124	3.155243	-0.22717	6.537659	-2.01772	8.328201
125	3.211529	-0.17089	6.593945	-1.96143	8.384488
126	3.246652	-0.13576	6.629068	-1.92631	8.419611
127	3.321836	-0.06058	6.704252	-1.85112	8.494795
128	3.459578	0.077162	6.841994	-1.71338	8.632537
129	3.511593	0.128673	6.894513	-1.66214	8.685323
130	3.579767	0.196847	6.962687	-1.59396	8.753497
131	3.642407	0.259487	7.025327	-1.53132	8.816137
132	3.715523	0.332603	7.098444	-1.45821	8.889253
133	3.805001	0.422081	7.187921	-1.36873	8.978731
134	3.872058	0.489103	7.255013	-1.30172	9.045841
135	3.943341	0.560387	7.326296	-1.23044	9.117124
136	4.013177	0.630222	7.396132	-1.16061	9.18696

Table 9 - Predictions from ARIMA(1,0,0)(1,0,0)[5] with drift - This table contains the detailed predictions plotted in Figure 33 along with their 80 percent and 95 percent prediction intervals that were shown in the dark blue and light blue areas respectively.

Predicted vs Actual:

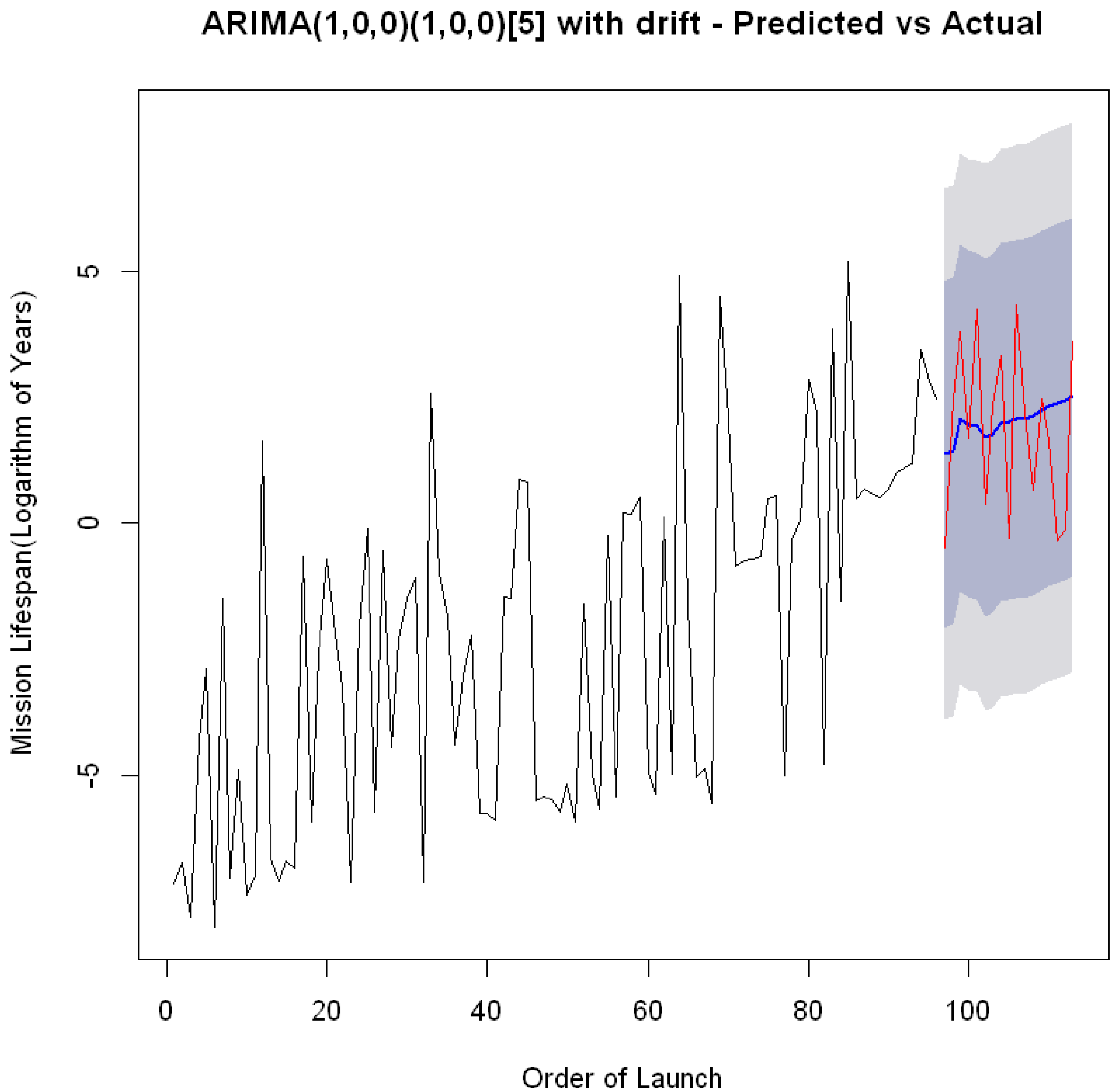

Figure 34 - ARIMA(1,0,0)(1,0,0) with drift - Predicted vs Actual - 15 percent of the original data is withheld and tested against a model generated with the remaining 85 percent. All withheld values fall within the dark blue area indicating that the model predicts the actual values reasonably well. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The withheld data is shown in red and the forecast generated with the model is shown in blue. 80 percent of projected values fall within the dark blue shaded are and 95 percent of all projected values fall within the light blue shaded area.

Diagnostics:

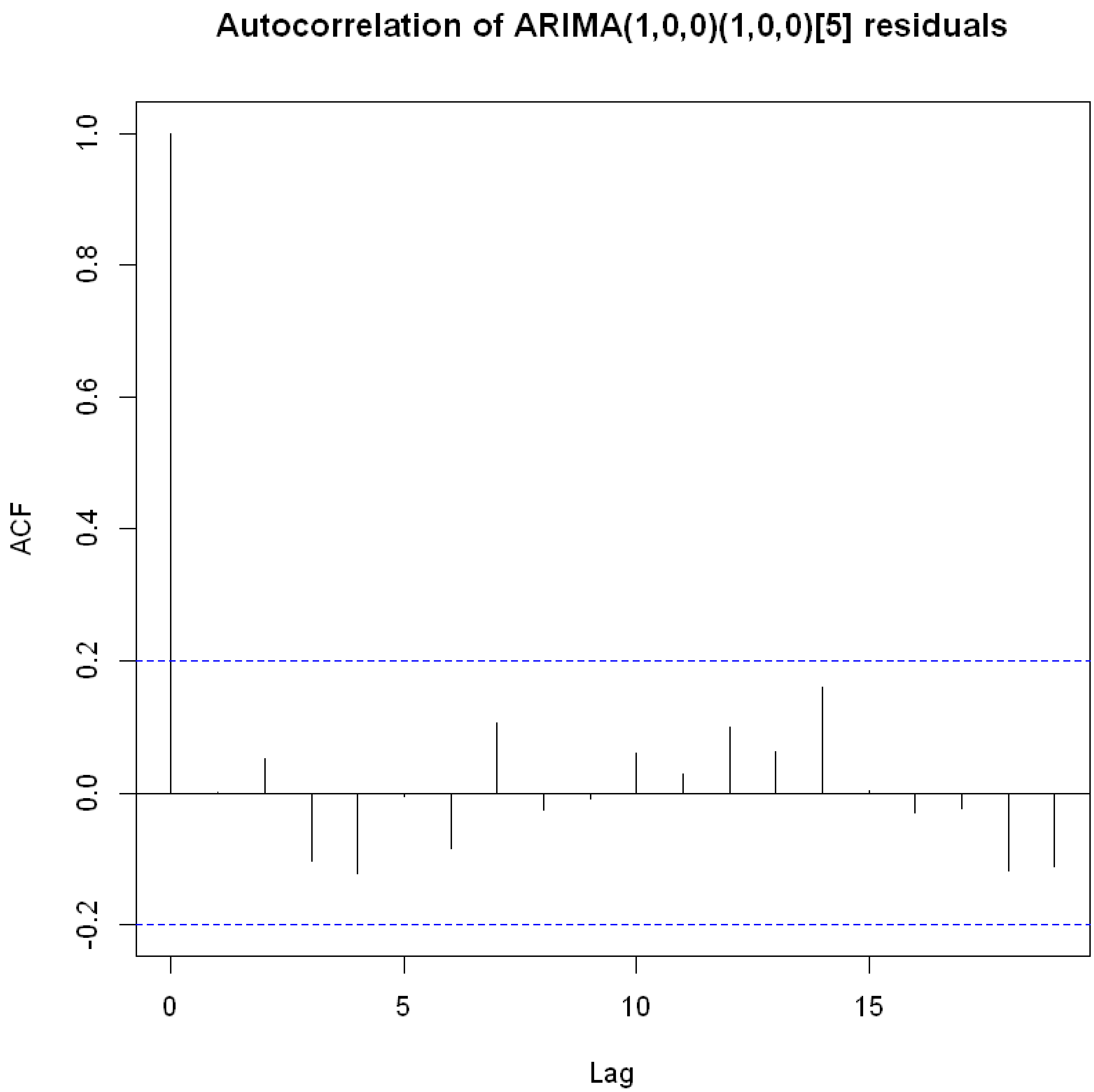

Figure 35 - Autocorrelation of ARIMA(1,0,0)(1,0,0)[5] Residuals – Unlike the ACF plot in Figure 20, this plot lacks significant autocorrelation and lag 5 which indicates better model quality. This also indicates seasonality since the model incorporates a 5-period seasonal term.

ARIMA(1,0,0)(1,0,0)[5]			
Predicted	Actual	Error	Squared Error
1.366264	-0.49989	1.866153	3.482528
1.42482	2.44369	1.01887	1.038097
2.065741	3.799628	1.733887	3.006363
1.960117	1.683646	0.276471	0.076436
1.922775	4.227464	2.304689	5.311592
1.694314	0.369321	1.324993	1.755608
1.764574	2.330355	0.565782	0.320109
1.985549	3.319869	1.33432	1.78041
2.013319	-0.29803	2.311353	5.342351
2.05876	4.315931	2.257171	5.094821
2.054739	2.188834	0.134095	0.017981
2.128027	0.647592	1.480435	2.191689
2.240321	2.441586	0.201265	0.040508
2.302613	1.655872	0.64674	0.418273
2.369478	-0.3545	2.723981	7.420075
2.423543	-0.12559	2.549137	6.498099
2.497615	3.600527	1.102912	1.216415
Mean Squared Error			2.647727

Table 10 - ARIMA(1,0,0)(1,0,0)[5] Mean Square Error - Each prediction generated by the model in Figure 29 is compared to the actual value. Each error is then squared, and the arithmetic mean of each of the squared errors is computed. The result shows an average squared error of 2.647727. The result indicates that the addition of a seasonal term significantly improves forecast accuracy.

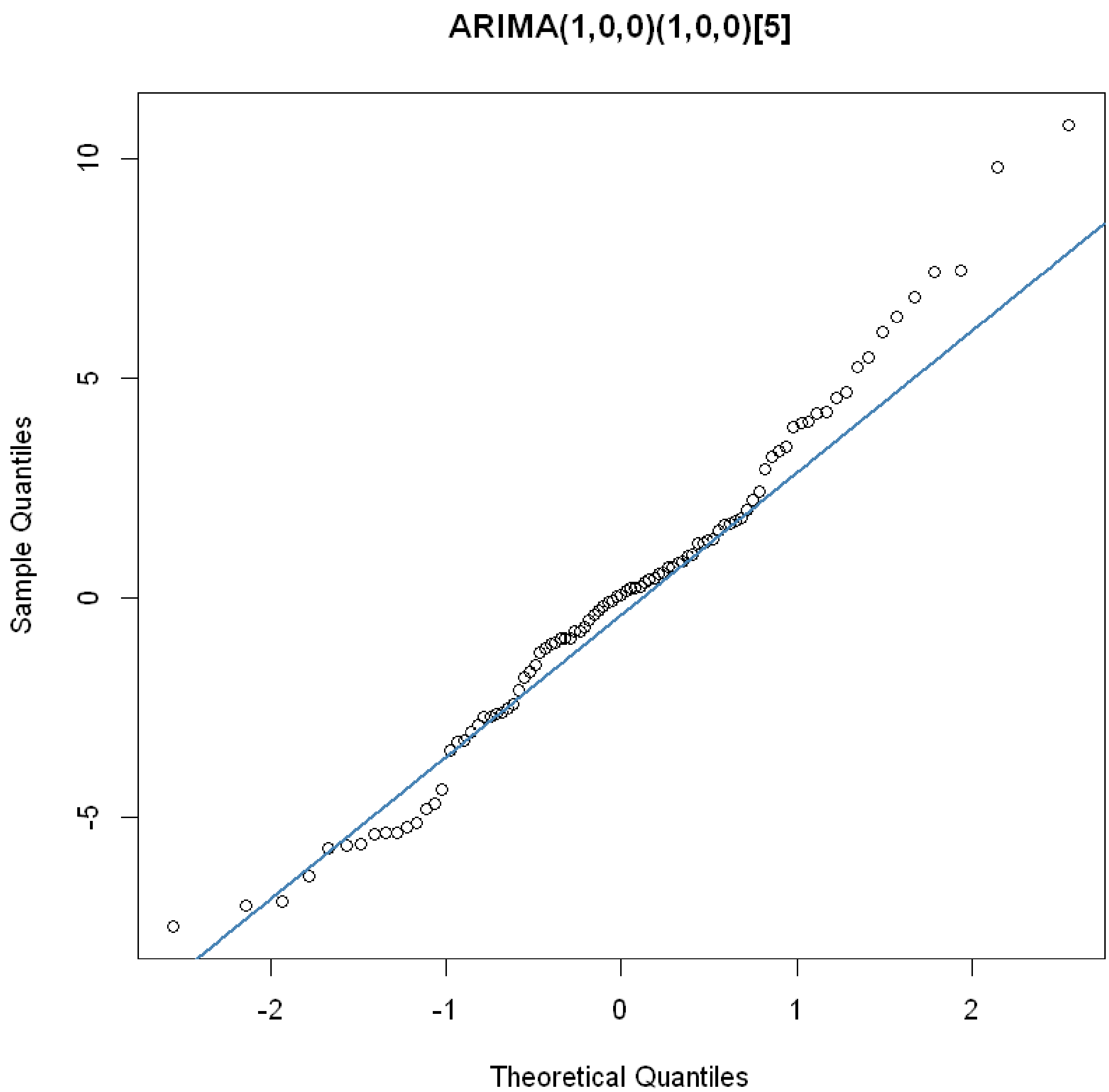

Figure 36 - QQ Plot for ARIMA(1,0,0)(1,0,0)[5] – Compare to Figure 21. The QQ Plot for the seasonal version of the ARIMA(1,0,0) model is noticeably straighter which indicates more normally distributed residuals and better model quality.

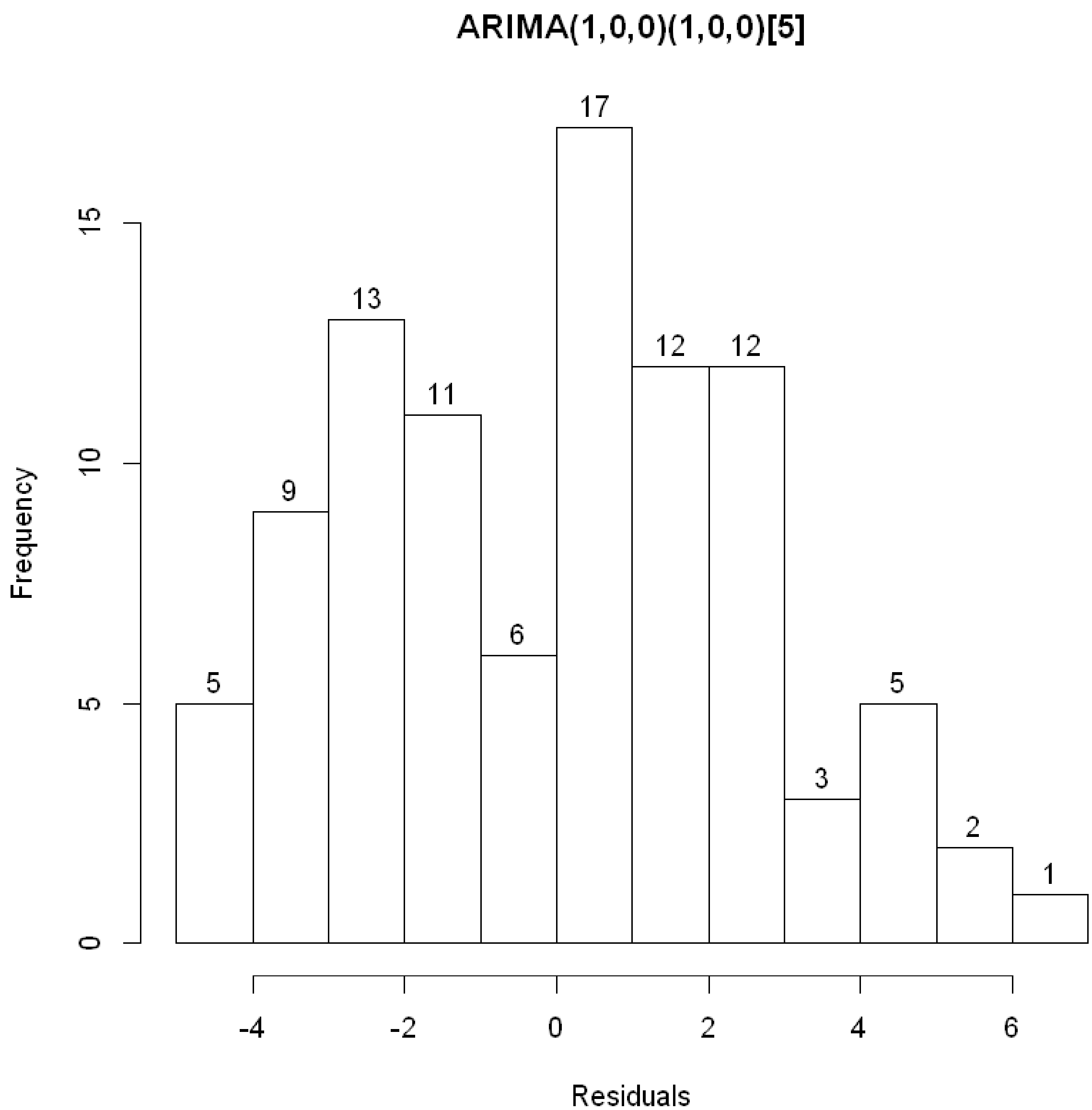

Figure 37 - Histogram of ARIMA(1,0,0)(1,0,0)[5] Residuals – Like the non-seasonal counterpart of this mode, the histogram of the residuals displays a significant skew in the negative bins, peaking at approximately -2. This shows that while this model is better than its non-seasonal version it is still missing significant information.

ARIMA(2,0,0)(1,0,0)[5]

The seasonal version of ARIMA(2,0,0) has slightly worse AIC and BIC scores than the seasonal version of ARIMA(1,0,0). However, it does have the worst Mean Square Error.

Forecast:

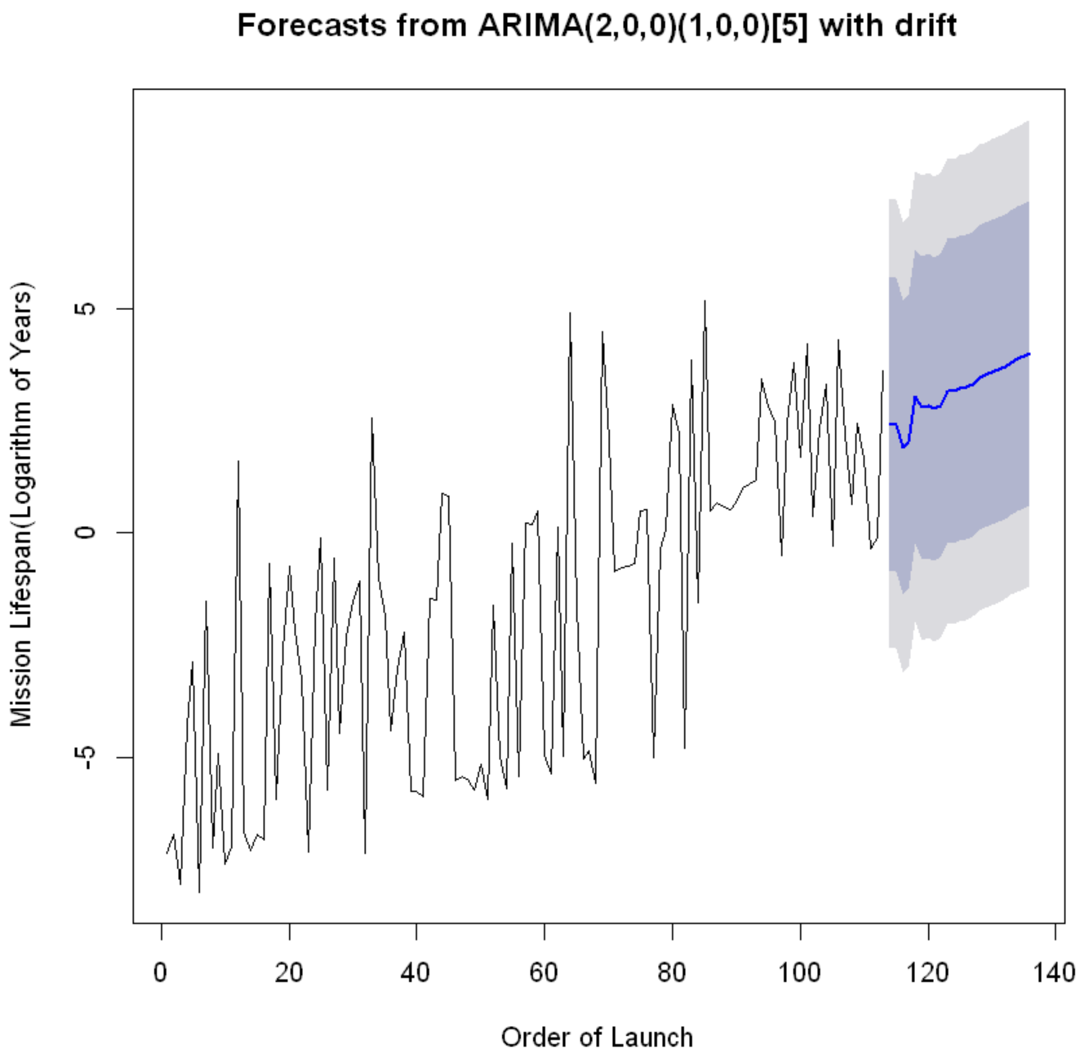

*Figure 38 - Forecasts from ARIMA(2,0,0)(1,0,0)[5] with drift - This graph shows predictions for lifespan values of future launches. The predictions show a clear increasing trend. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The dark blue shaded area shows where 80 percent of possible future values lie. The light blue shaded area shows where 95 percent of possible values lie. A linear regression is fitted and an ARIMA model is applied to the residuals. The non-seasonal portion is Current Value = (Lag 1 Value) * 0.0062 + (Lag 2 Value) * 0.0316. The seasonal portion is $Current\ Value_5$ = (Lag_5) * 0.2598, the subscript signifying the seasonal period. The 1-step ahead forecast is: Forecast = (Non – Seasonal Forecast) * (Seasonal Forecast) + (Ordinality * 0.0724) – 5.8315.*

23 point forecast for ARIMA(2,0,0)(1,0,0)[5] with drift					
	Point Forecast	Lo.80	Hi.80	Lo.95	Hi.95
114	2.455046829	-0.823584274	5.733678	-2.559186739	7.46928
115	2.422817015	-0.855877894	5.701512	-2.591514137	7.437148
116	1.902292317	-1.378042199	5.182627	-3.114546398	6.919131
117	2.018782666	-1.261552105	5.299117	-2.998056438	7.035622
118	3.038772039	-0.241564376	6.319108	-1.97806958	8.055614
119	2.794899163	-0.594227678	6.184026	-2.388323055	7.978121
120	2.840085645	-0.549045527	6.229217	-2.343143196	8.023314
121	2.758472851	-0.630765416	6.147711	-2.424919778	7.941865
122	2.842343758	-0.546894526	6.231582	-2.341048897	8.025736
123	3.160933371	-0.228305021	6.550172	-2.022459449	8.344326
124	3.151188225	-0.245266584	6.547643	-2.043241153	8.345618
125	3.216536914	-0.179918187	6.612992	-1.97789291	8.410967
126	3.248944831	-0.147517483	6.645407	-1.945496024	8.443386
127	3.324343218	-0.072119097	6.720806	-1.870097638	8.518784
128	3.460718407	0.064256086	6.857181	-1.73372246	8.655159
129	3.511796587	0.114847788	6.908745	-1.683388282	8.706981
130	3.582383147	0.185434329	6.979332	-1.612801752	8.777568
131	3.644412132	0.247462828	7.041361	-1.550773511	8.839598
132	3.717609471	0.320660166	7.114559	-1.477576173	8.912795
133	3.806647773	0.409698468	7.203597	-1.388537871	9.001833
134	3.873527044	0.47654491	7.270509	-1.321708808	9.068763
135	3.945474334	0.548492198	7.342456	-1.24976152	9.14071
136	4.015198479	0.618216311	7.412181	-1.180037425	9.210434

Table 11 - Predictions from ARIIMA(2,0,0)(1,0,0)[5] with drift - This table contains the detailed predictions plotted in Figure 38 along with their 80 percent and 95 percent prediction intervals that were shown in the dark blue and light blue areas respectively.

Predicted vs Actual:

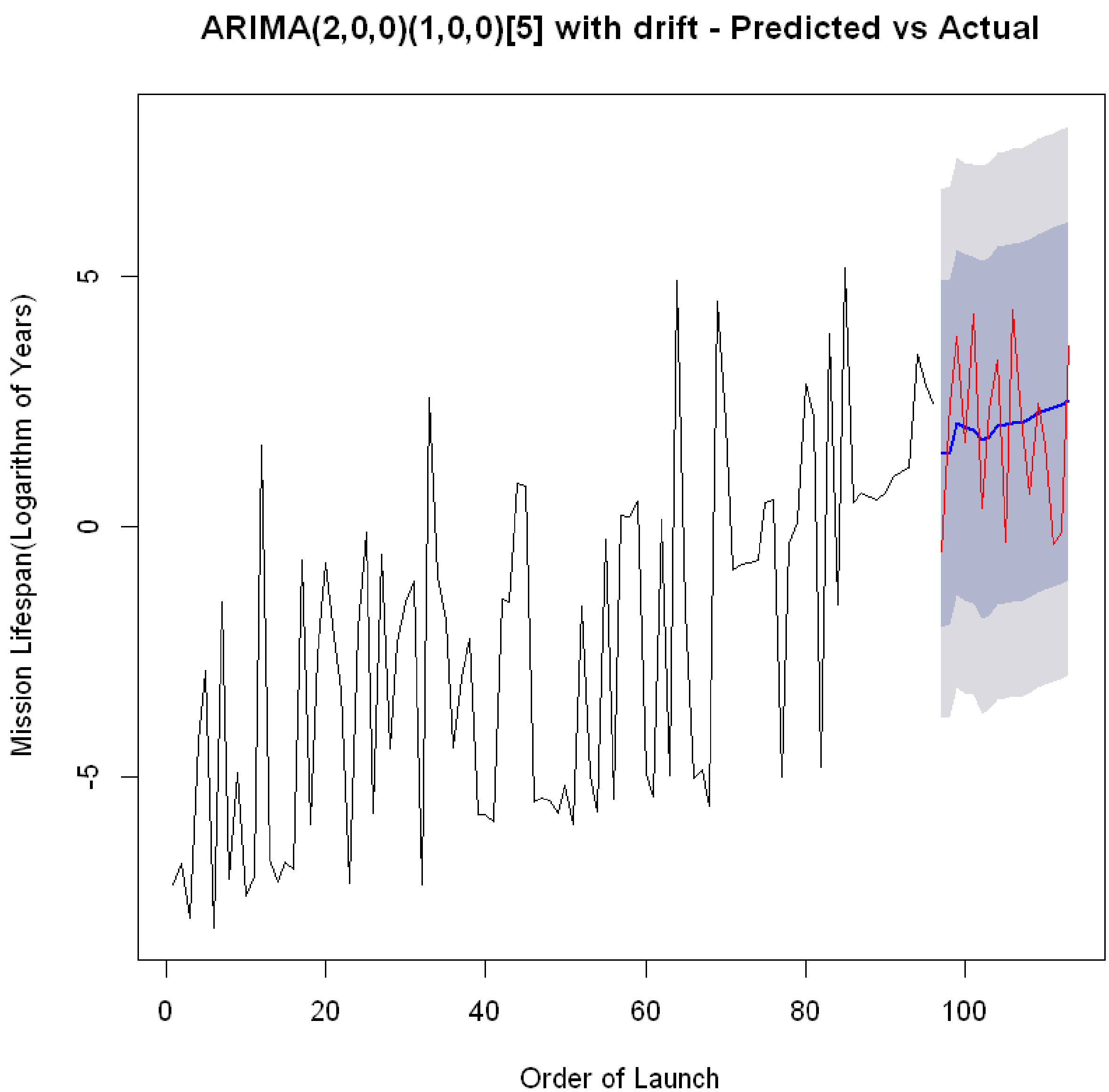

Figure 39 - ARIMA(2,0,0)(1,0,0)[5] - Predicted vs Actual - 15 percent of the original data is withheld and tested against a model generated with the remaining 85 percent. All withheld values fall within the dark blue area indicating that the model predicts the actual values reasonably well. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The withheld data is shown in red and the forecast generated with the model is shown in blue. 80 percent of projected values fall within the dark blue shaded are and 95 percent of all projected values fall within the light blue shaded area.

Diagnostics:

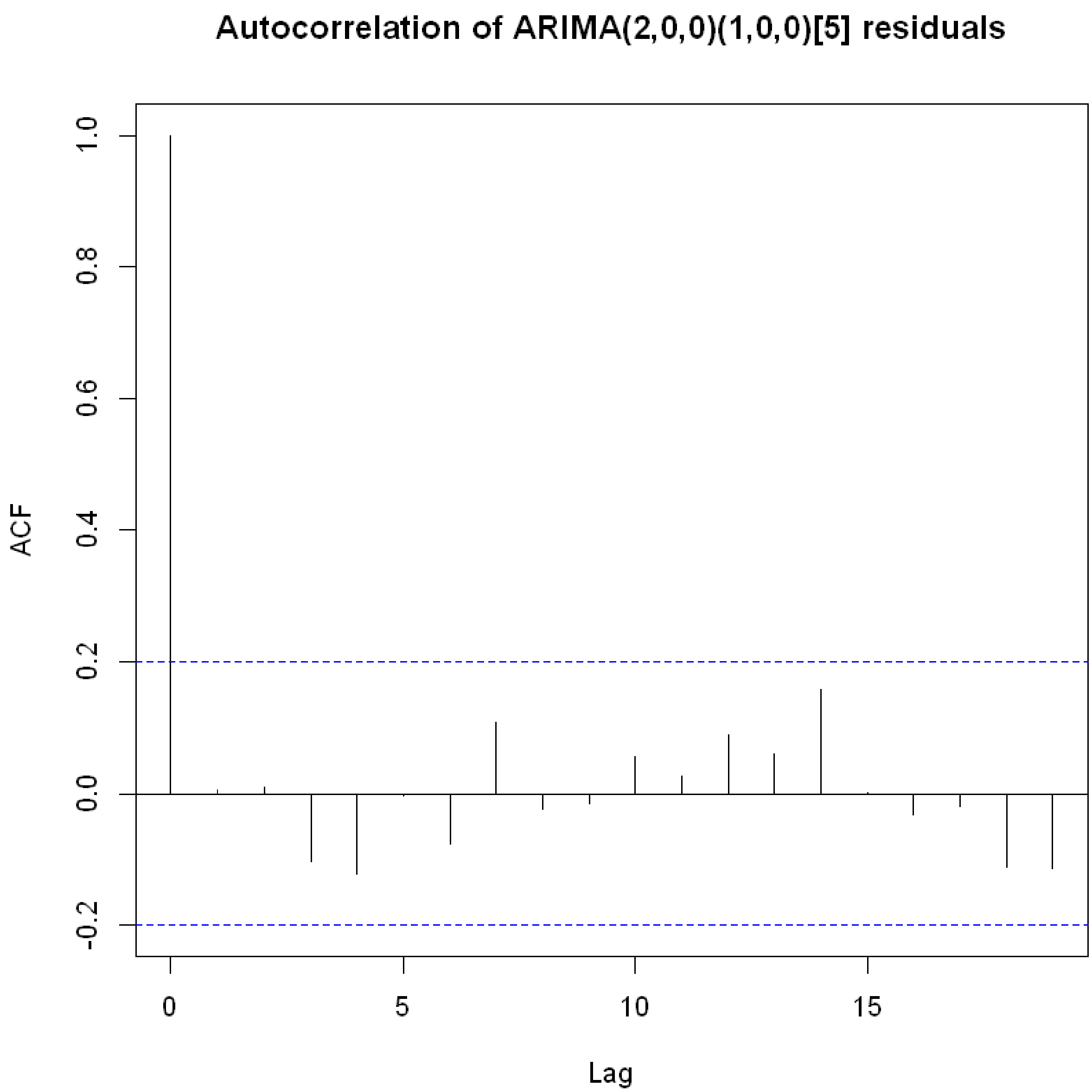

Figure 40 - Autocorrelation of ARIIMA(2,0,0)(1,0,0)[5] residuals – The ACF plot shows a lack of autocorrelation among the residuals, which was also seen in Figure 35. There is also no significant correlation at lag 5 which indicates that this model performs better than its non-seasonal version.

ARIMA(2,0,0)(1,0,0)[5]			
Predicted	Actual	Error	Squared Error
1.451732	-0.49989	1.951622	3.808829
1.489324	2.44369	0.954366	0.910815
2.076553	3.799628	1.723075	2.968987
1.969291	1.683646	0.285645	0.081593
1.930222	4.227464	2.297241	5.277318
1.725275	0.369321	1.355954	1.83861
1.7902	2.330355	0.540156	0.291768
1.996771	3.319869	1.323099	1.75059
2.024505	-0.29803	2.322539	5.394189
2.069806	4.315931	2.246125	5.045076
2.072391	2.188834	0.116443	0.013559
2.144472	0.647592	1.49688	2.240649
2.253028	2.441586	0.188558	0.035554
2.315532	1.655872	0.659659	0.435151
2.382559	-0.3545	2.737063	7.491512
2.438587	-0.12559	2.564181	6.575023
2.51251	3.600527	1.088017	1.18378
Mean Squared Error			2.667236

Table 12 - ARIMA(2,0,0)(1,0,0) Mean Square Error - Each prediction generated by the model in Figure 29 is compared to the actual value. Each error is then squared, and the arithmetic mean of each of the squared errors is computed. The result shows an average squared error of 2.667236. The result indicates that the addition of a seasonal term significantly improves forecast accuracy.

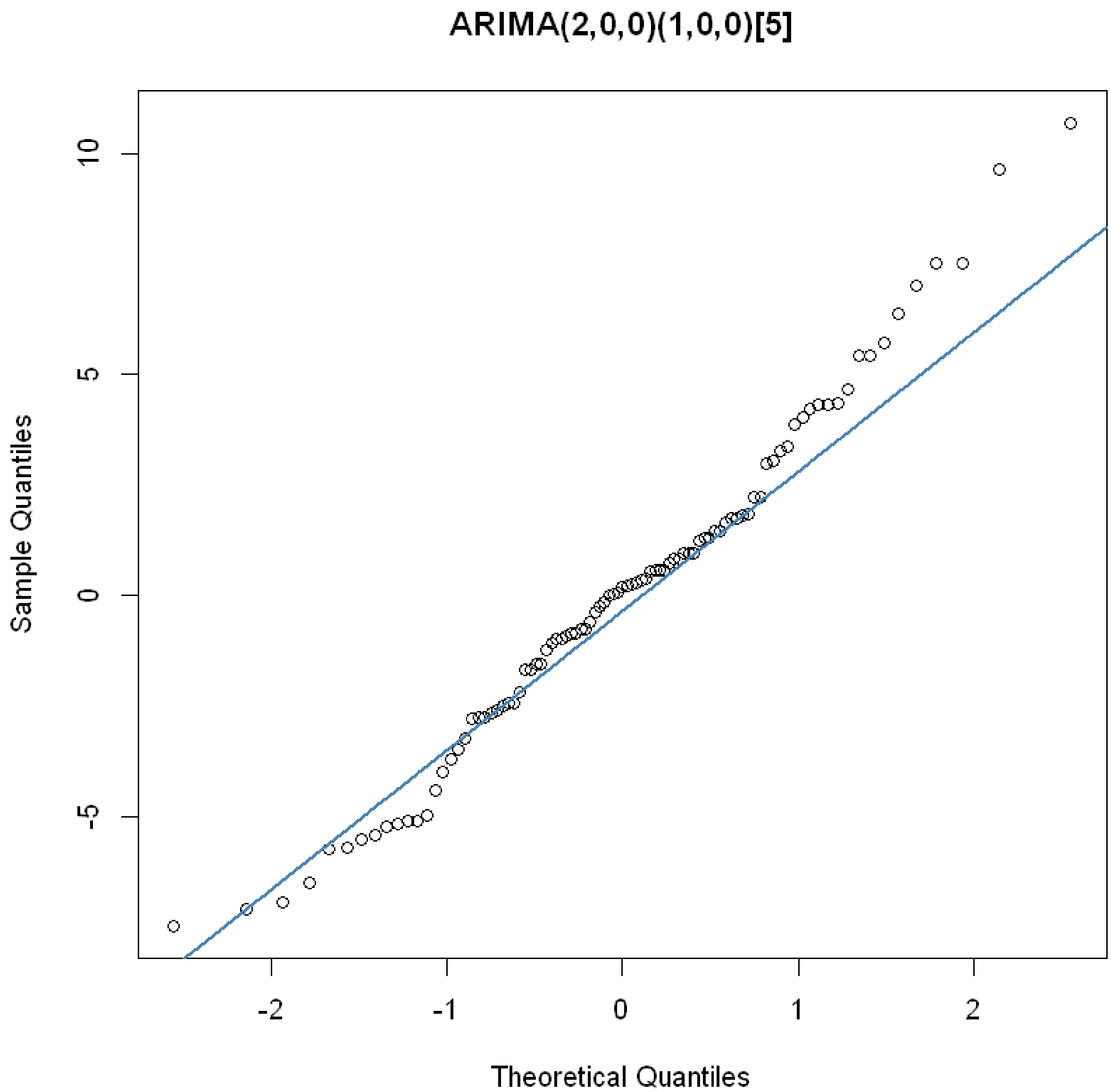

Figure 41 - QQ Plot for ARIMA(2,0,0)(1,0,0)[5] – This QQ Plot is not noticeably different from Figure 36 but is significantly straighter than those of non-seasonal models which indicates better model quality.

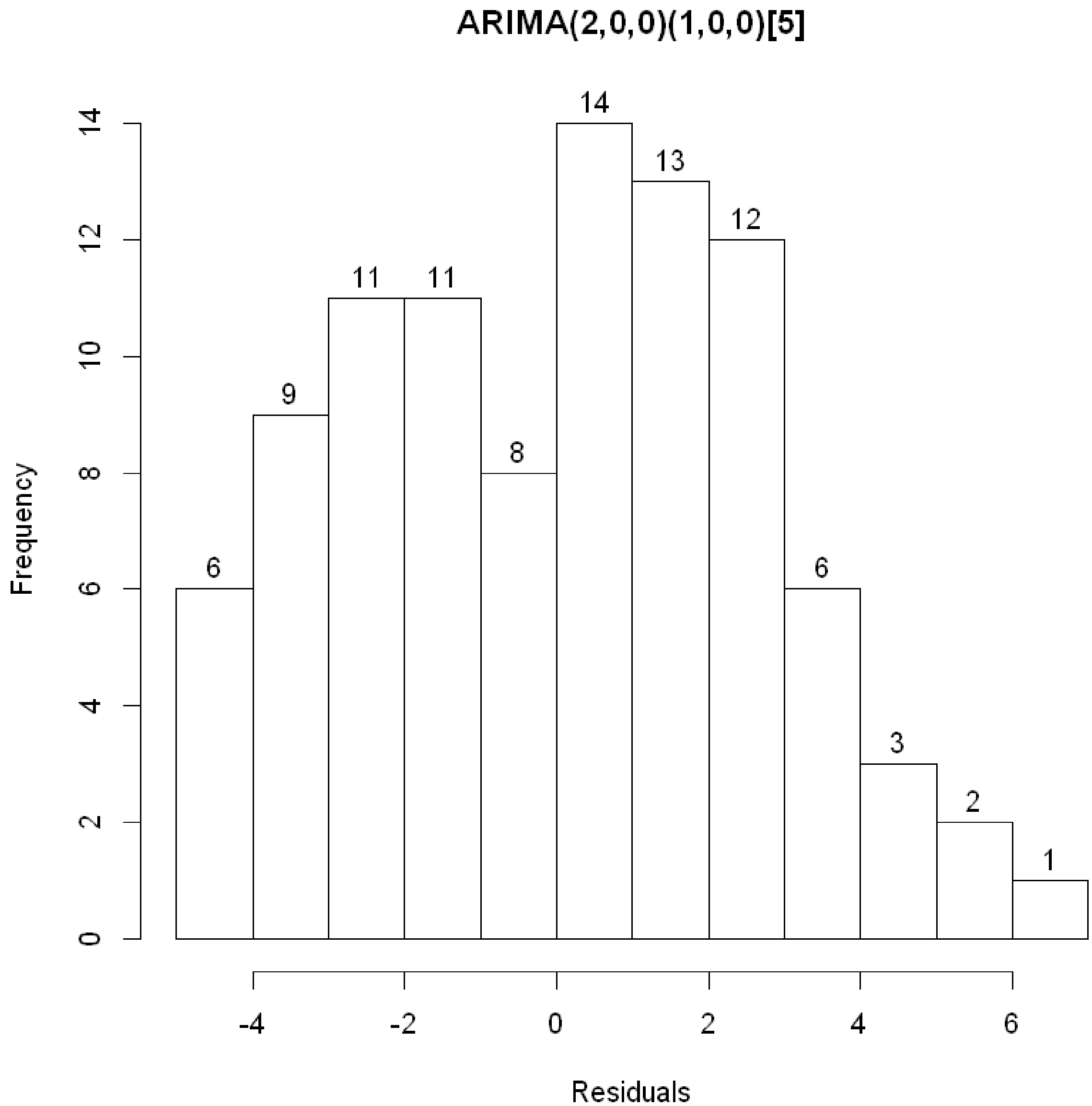

Figure 42 - Histogram of Residuals ARIMA(2,0,0)(1,0,0)[5] – The histogram of residuals displays significant skewing at -2 as with Figure 37. This skewing indicates that significant information is being missed in the model.

ARIMA (1,0,1)(1,0,0):

The seasonal version of ARIMA(1,0,1) has the best Mean Square Error of any model, seasonal or non-seasonal. This model has the worst AIC and BIC of any of the seasonal models, making it the least parsimonious.

Forecast:

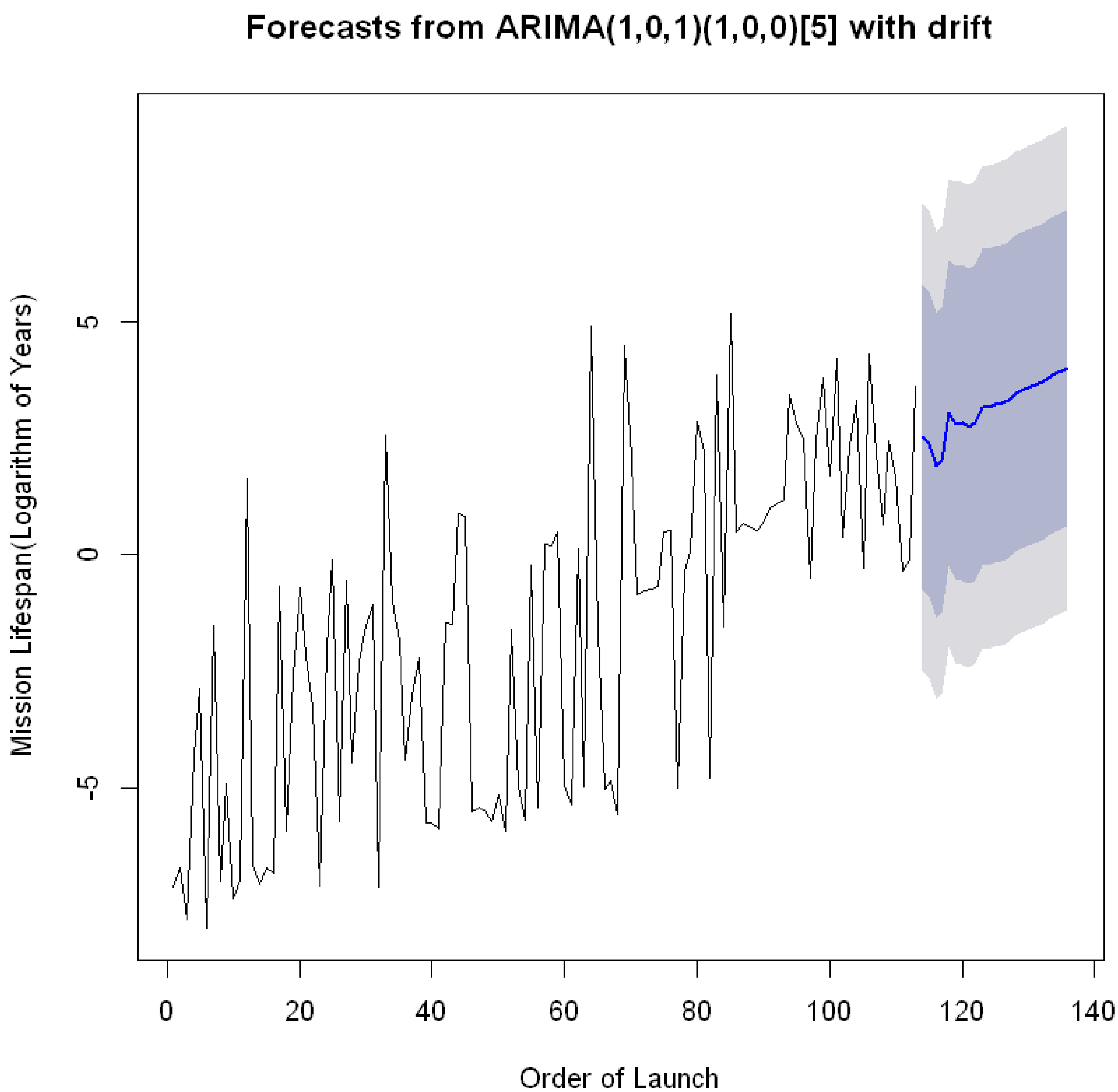

*Figure 43 - Forecasts from ARIMA(1,0,1)(1,0,0)[5] with drift - This graph shows predictions for lifespan values of future launches. The predictions show a clear increasing trend. The x-axis shows the order of launch and the y-axis shows the logarithm of mission lifespan. The dark blue shaded area shows where 80 percent of possible future values lie. The light blue shaded area shows where 95 percent of possible values lie.A linear regression is fitted and an ARIMA model is applied to the residuals. The non-seasonal portion is Current Value = (Lag 1 Value) * - 0.6521 + (Lag 1 Residual) * (0.6523). The seasonal portion is* $Current\ Value_5$ *= (*Lag_5*) * -0.2604, the subscript signifying the seasonal period. The 1-step ahead forecast is: Forecast = (Non – Seasonal Forecast) * (Seasonal Forecast) + (Ordinality * 0.0724) – 5.8276.*

23 point forecast from ARIMA(1,0,1)(1,0,0)[5] with drift					
	Point Forecast	Lo.80	Hi.80	Lo.95	Hi.95
114	2.522765	-0.75758	5.803107	-2.49409	7.539615
115	2.37113	-0.90921	5.651472	-2.64572	7.38798
116	1.901644	-1.3787	5.181987	-3.11521	6.918495
117	2.014519	-1.26582	5.294861	-3.00233	7.03137
118	3.038336	-0.24201	6.318678	-1.97851	8.055187
119	2.811159	-0.57857	6.200887	-2.37298	7.9953
120	2.825285	-0.56444	6.215012	-2.35886	8.009426
121	2.756539	-0.63319	6.146266	-2.4276	7.94068
122	2.839491	-0.55024	6.229218	-2.34465	8.023631
123	3.159561	-0.23017	6.549288	-2.02458	8.343701
124	3.15396	-0.24305	6.550974	-2.04132	8.349244
125	3.211163	-0.18585	6.608177	-1.98412	8.406447
126	3.246804	-0.15021	6.643817	-1.94848	8.442088
127	3.321931	-0.07508	6.718944	-1.87335	8.517215
128	3.458799	0.061785	6.855812	-1.73649	8.654083
129	3.510873	0.113366	6.90838	-1.68517	8.706911
130	3.5793	0.181793	6.976807	-1.61674	8.775339
131	3.642112	0.244606	7.039619	-1.55393	8.838151
132	3.715206	0.317699	7.112713	-1.48083	8.911244
133	3.804374	0.406867	7.201881	-1.39166	9.000412
134	3.871465	0.473925	7.269005	-1.32462	9.067555
135	3.942814	0.545274	7.340354	-1.25328	9.138904
136	4.012701	0.615161	7.410242	-1.18339	9.208791

Table 13 - Predictions from ARIMA(1,0,1)(1,0,0)[5] with drift - This table contains the detailed predictions plotted in Figure 43 along with their 80 percent and 95 percent prediction intervals that were shown in the dark blue and light blue areas respectively.

Predicted vs Actual:

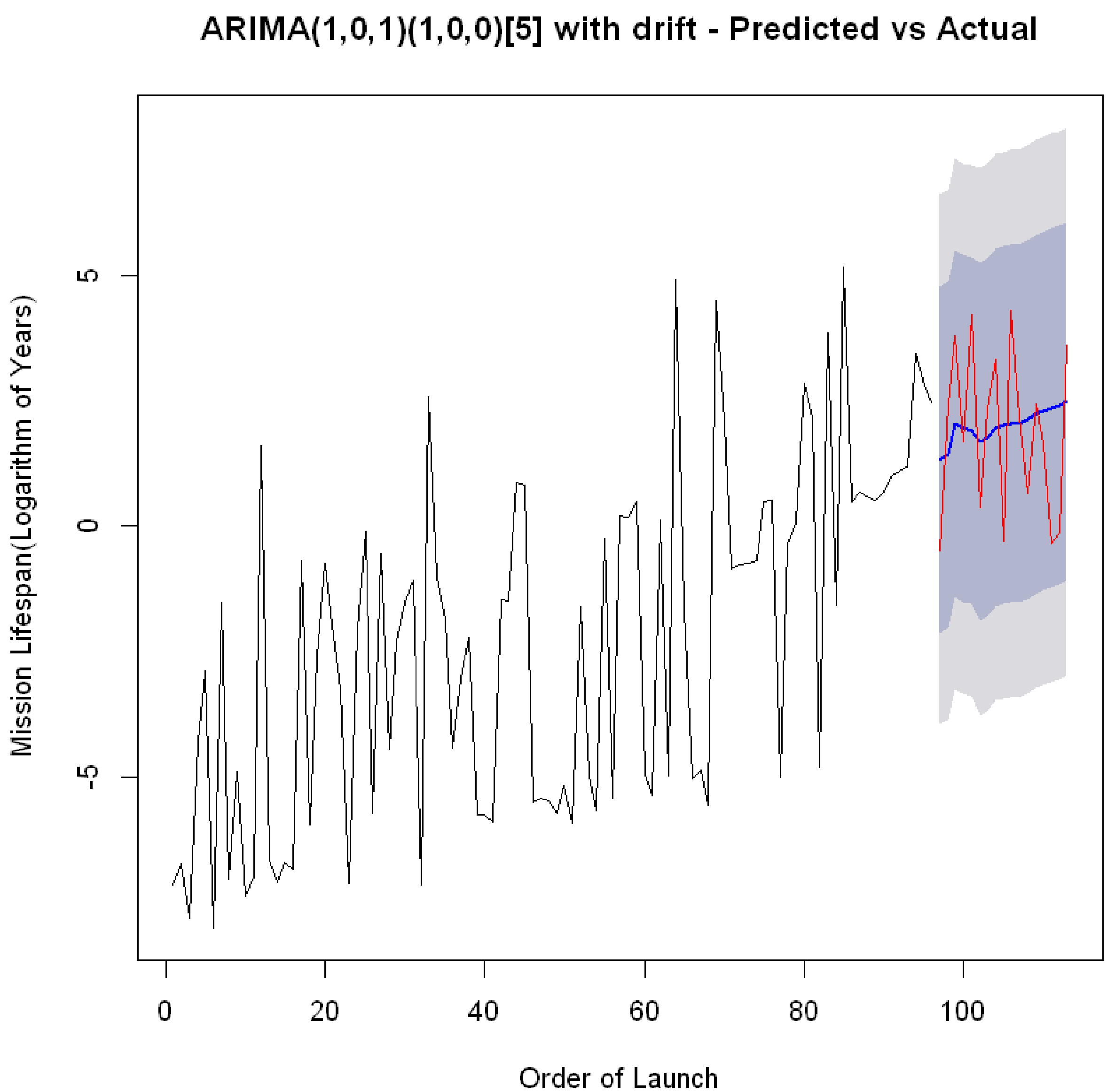

Figure 44 - ARIMA(1,0,1)(1,0,0) with drift - Predicted vs Actual - 15 percent of the data is withheld and a model is generated with the remaining 85 percent. The withheld data is plotted in red and the predictions are plotted in blue. The dark blue area represents where 80 percent of all projected values lie and the light blue area represents where 95 percent of all projected values lie.

Diagnostics:

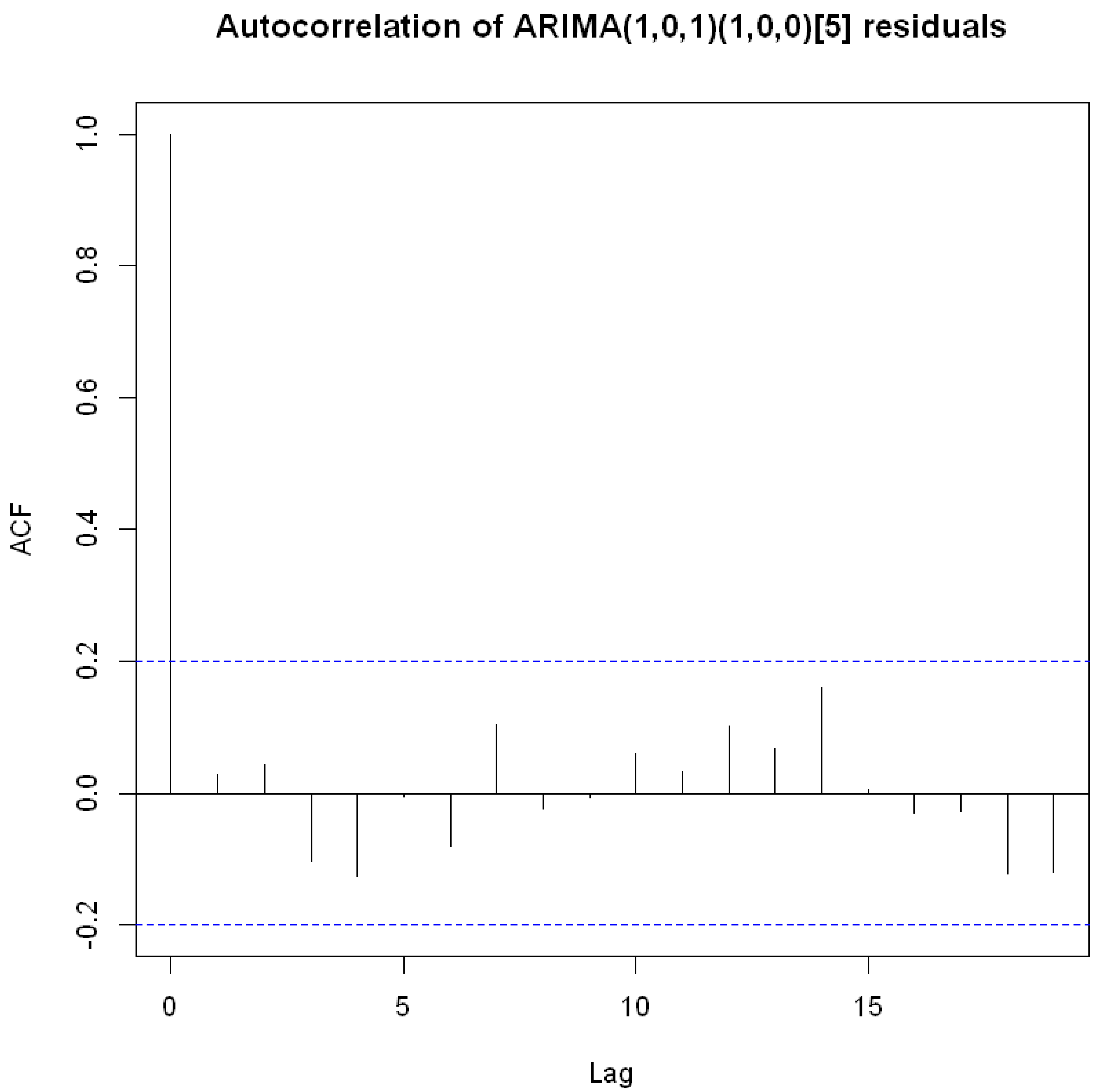

Figure 45 - Autocorrelation of ARIMA(1,0,1)(1,0,0)[5] Residuals – Like every other autocorrelation plot for seasonal models, this lacks a significant correlation at lag 5

ARIMA(1,0,1)(1,0,0)[5]			
Predicted	Actual	Error	Squared Error
1.332928	-0.49989	1.832817	3.35922
1.426359	2.44369	1.017331	1.034963
2.050443	3.799628	1.749185	3.059647
1.951961	1.683646	0.268315	0.071993
1.913999	4.227464	2.313465	5.35212
1.682948	0.369321	1.313627	1.725617
1.761461	2.330355	0.568894	0.32364
1.976157	3.319869	1.343712	1.805562
2.006245	-0.29803	2.304279	5.309701
2.052029	4.315931	2.263902	5.125252
2.048474	2.188834	0.140361	0.019701
2.123862	0.647592	1.47627	2.179373
2.233903	2.441586	0.207683	0.043132
2.296935	1.655872	0.641062	0.410961
2.363954	-0.3545	2.718457	7.39001
2.418409	-0.12559	2.544003	6.471954
2.492973	3.600527	1.107553	1.226675
Mean Squared Error			2.641737

Table 14 - ARIMA(1,0,1)(1,0,0)[5] Mean Square Error - Each prediction generated by the model in Figure 44 is compared to the actual value. Each error is then squared, and the arithmetic mean of each of the squared errors is computed. The result shows an average squared error of 2.641737.

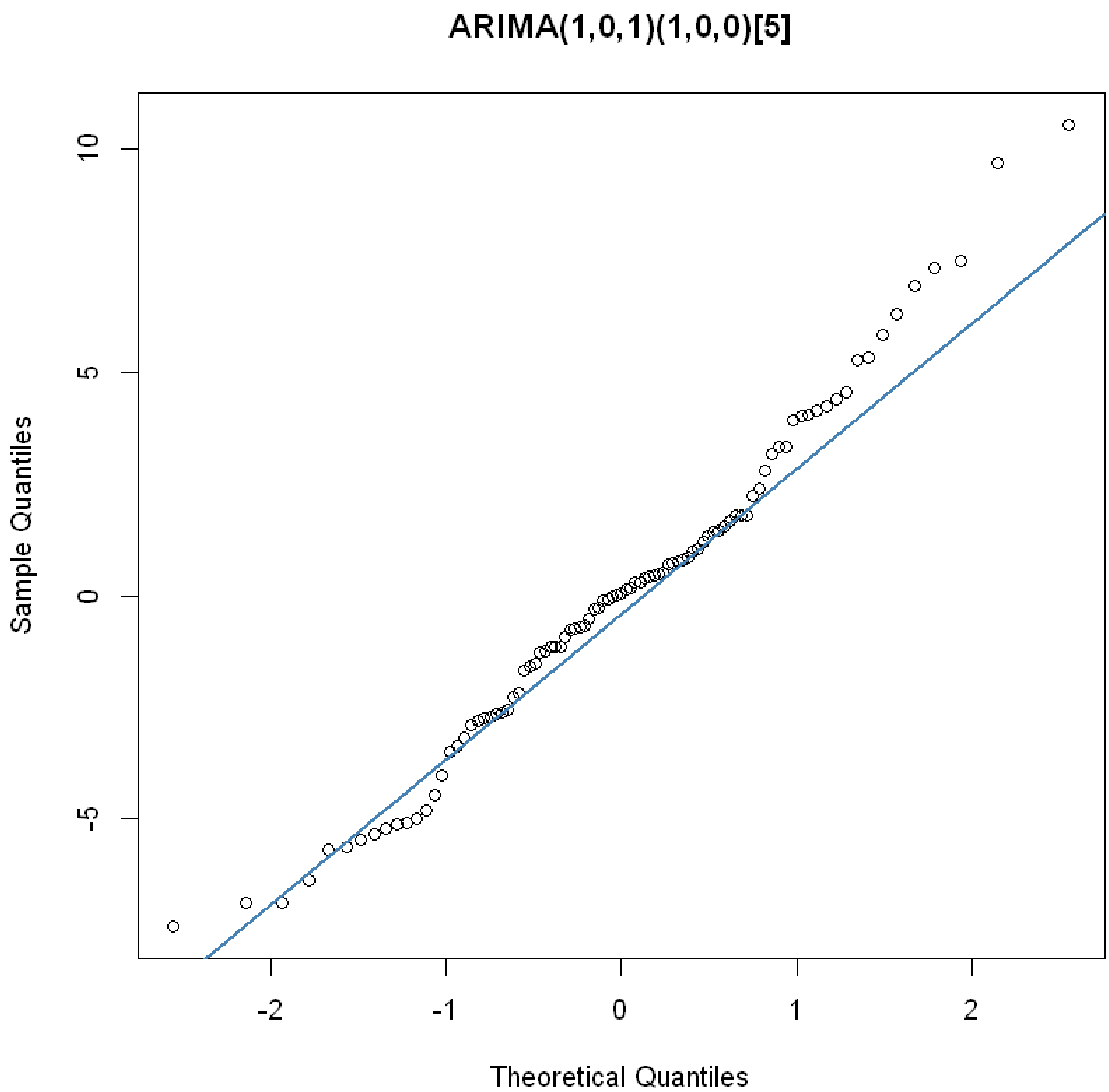

Figure 46 - QQ Plot for ARIMA(1,0,1)(1,0,0)[5] – This QQ Plot appears largely identical to the QQ Plots generated for ARIMA(1,0,0)(1,0,0)[5] and ARIMA(2,0,0)(1,0,0)[5].

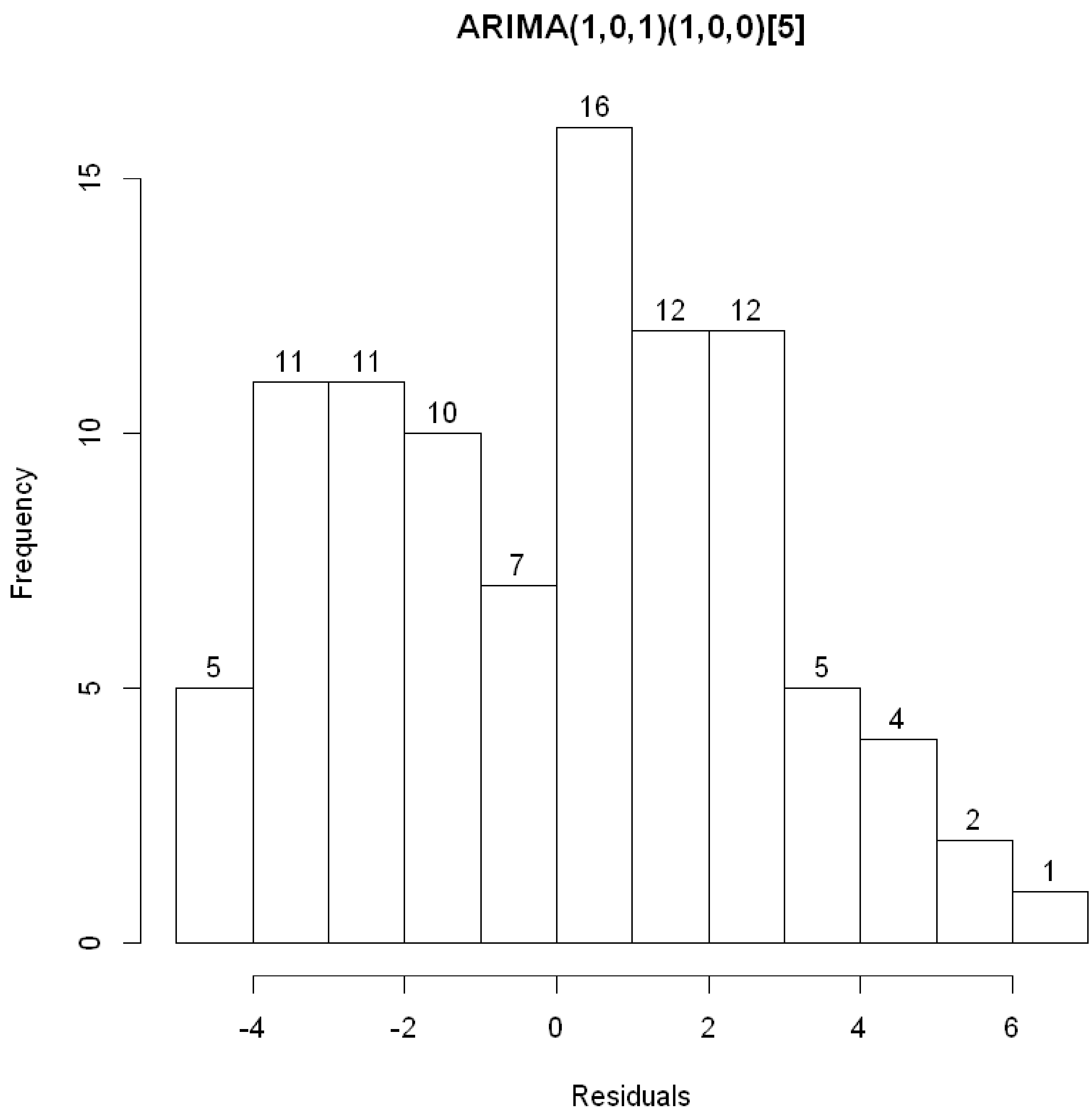

Figure 47 - Histogram of Residuals for ARIMA(1,0,1)(1,0,0)[5] – This histogram appears largely identical to the plot generated in Figure 42.

Appendix B: Paper for the Arkansas Academy of Science (in press)

Moore's Law and Space Exploration: New Insights and Next Steps

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Running title: Moore's Law and Space Exploration

Abstract

Understanding how technology changes over time is important for industry, science, and government policy. Empirical examination of the capability of technologies across various domains reveals that they often progress at an exponential rate. In addition, mathematical models of technological development have proven successful in deepening our understanding. One area that has not been shown to demonstrate exponential trends, until recently, has been space travel.

This paper will present plots illustrating trends in the mean lifespan of satellites whose lifespans ended in a given year. Our study identifies both Wright's law and Moore's law regressions. For the Moore's law regression, we found a doubling time of approximately 15 years. For Wright's law we can see an approximate doubling of lifespan with every doubling of accumulated launches. We conclude by presenting a conundrum generated by the use of Moore's law that is the subject of ongoing research.

Introduction

It has been observed that the rates of increase of technological capability in a variety of domains often follow exponential trends. For such domains there is a fairly predictable time constant at which the capability of the technology doubles although the time constants themselves vary quite a bit across domains (Magee *et al*. 2014). These trends are exponential and often described as conforming to "Moore's law," which originally described how the number of components that can be built into an integrated circuit doubles approximately every 18 months (Moore 1965).

But what causes these exponential patterns? Some noteworthy research done in this area suggests that this exponential progress is due to innovators applying lessons and principles from one domain to another domain (Basnet and Magee 2016; Arthur and Polak 2006; Axtell *et al.* 2013). The newly generated ideas will then be available for use in another domain and so on. The complexity of the technological system itself as well as functional requirements of the system influence how quickly the technology can be improved and leads to differing rates of progress (McNerny *et al*. 2011; Basnet and Magee 2016; Basnet and Magee 2017).

Another important description of technological progress was discovered by the engineer Theodore Paul Wright. This principle, known as "Wright's law," describes how as the volume produced of a manufactured good increases, the per-unit cost of the good falls at a predictable rate (Wright 1936). While Wright's law has important implications for operations management and business strategy it has also proven useful for technology foresight. An influential report indicates an equivalence between Wright's law and Moore's law when volume produced increases exponentially over time (Sahal 1979). A study in 2013 further compared Moore's law and Wright's law (Nagy *et al*. 2013).

While such patterns have been observed for fields as diverse as genome sequencing, LEDs, and 3D printing, they have not been observed for space travel. In fact, it is widely held that progress in space travel "has stalled" (Hicks 2015). A primary focus of our research program has been to determine if we are in a "space winter" or if there are in fact exponential trends to be found (Berleant *et al*. 2017).

The question of how to measure progress is not an easy one to answer. In fact, the wrong choice of metric may obscure the fact that space travel is improving (Roberts 2011). Cost may show an improvement trend, but collecting and analyzing the required data has proven non-trivial. As an alternative approach, evidence has been found suggesting an exponential trend with regards to spacecraft lifespan (Berleant *et al*. 2019). A key question (Nagy *et al.* 2013) has been whether this trend best fits a Moore's law-like pattern (improvement with respect to time) or a Wright's law-like pattern (improvement with respect to accumulated production volume).

One reason given for the apparent lackluster progress of space technology is the lack of commercialization. Matt Ridley in his book *The Rational Optimist* and others make the case that financial incentives play an important role in the development of any technology. From British capital markets during the industrial revolution to venture capitalists on Sandhill Road in Silicon Valley, history gives us good reason to believe that the expectation of profit is a strong driver of technological progress. Satellite technology represents the most commercialized aspect of space technology today. For this reason, we hypothesized that a data analysis of satellite technology may provide indications of exponential trending.

Analysis of Satellite Data

Figure 1 shows the mean lifespan of all satellites whose lifespans ended in a given year. A more detailed discussion of the data appears in Berleant *et al*. (2019), while here we emphasize those aspects most salient to (1) the focus of the present article, and (2) those elements of Figure 1 that represent an advance on the analogous figure in Berleant *et al*. (2019). The Moore's law regression is provided in equation (1) and the Wright's law regression is provided in equation (8).

The top curve, Annual Count, shows the number of satellites whose lifespans ended (not launched) in the year given on the x-axis.

Both the Wright's law and Moore's law regressions show a general upward trajectory. Wright's law displays some irregular variations when plotted with respect to time, which is to be expected since Wright's law defines volume produced as the independent variable and not the x-axis variable, passage of time. If the x-axis showed volume instead, the regression curve would be free of such variations (but the Moore regression would then have them). For the Moore's law regression we have a doubling time of approximately 15 years. For Wright's law we can see an approximate doubling of lifespan with every doubling of accumulated launches.

Some of the earliest years were discarded from both regressions due to their inclusion leading to a poor fit to the regression curves. While this may seem contrary to the point of doing a regression it is useful for maintaining the ability of the model to predict, when early data is outlying or seemingly anomalous, and the primary interest is in extrapolating to the future. In this case, early launches were not representative of satellite technology as a whole and later data is more relevant than earlier data for the purpose of making predictions.

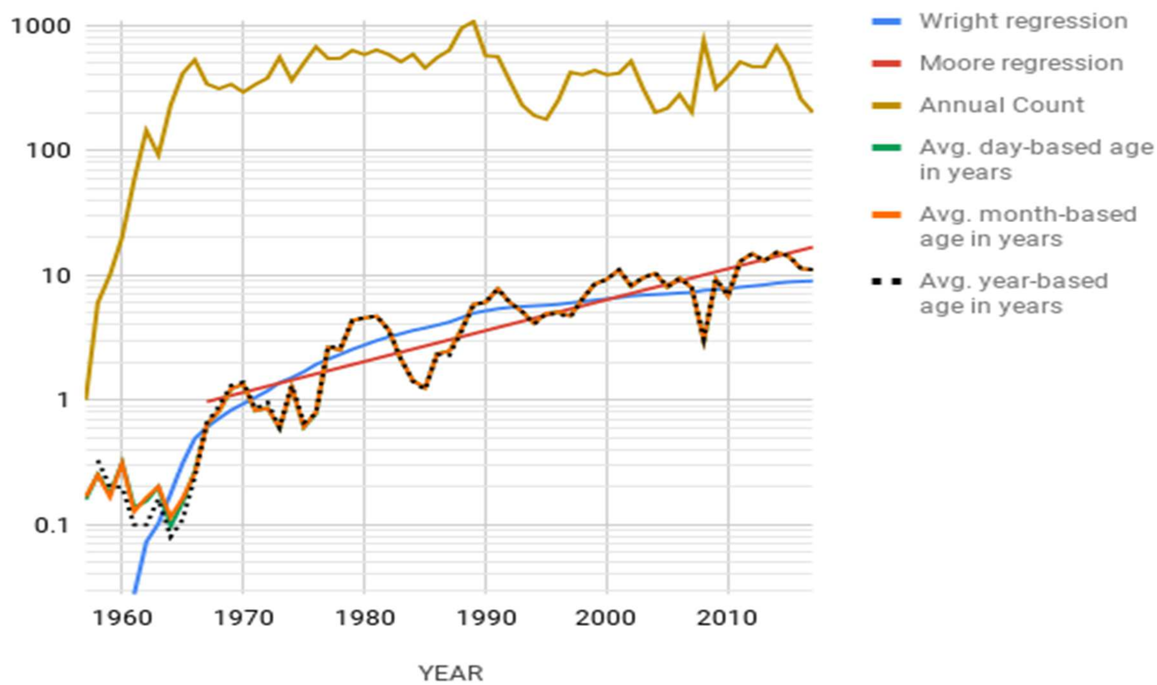


Figure 48. Annual count (top curve) and average age of satellites ending their lifespans each year.

An important question for measuring spacecraft lifespans is the correct unit of time to use. Lifespans were measured in days, months, and years (and then normalized so they could be directly compared) to examine how much using years and months distorted the graphs compared to more precise measurements in days. From Figure 1 it appears that years is not as good as months or days which are nearly identical. This occurs because measuring lifespan using years consists of subtracting the launch year from the end year. For example, suppose that a satellite was launched in December of 2016 and stopped functioning in March of 2017. Using years to measure this satellite's lifespan would give us a value of one year when in fact it had a lifespan of only three months.

End year was chosen rather than launch year because recently launched satellites would often still be in orbit, with only the shortest lived of them therefore contributing lifespan data for recent years, skewing the results and preventing a meaningful analysis.

Figure 2 illustrates an example of this phenomenon with lifespan data for spacecraft sent on deep space missions. When measured with respect to launch year we see that average lifespan increases until approximately 2000 and then decreases afterward, as significant numbers of craft launched in post-2000 years are still operational. This is because only short-lived spacecraft from this period are measured because these shorter-lived are the ones whose lifespans are available, leading to the noticeable decline in average lifespan due to the biased data, beginning in approximately 2000 and continuing to the present.

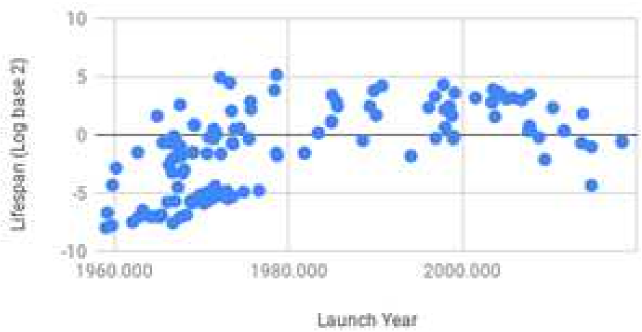


Figure 49. Lifespan vs. launch year for deep space craft.

The Moore's law conundrums

When comparing the RMS error of the Moore and Wright regressions it initially appears that Wright's law has a slightly better fit (Berleant *et al*. 2019). However, Wright's law may be more useful for another reason as well which isn't so obvious. While choosing the end year rather than start year made the analysis more feasible by removing the bias problem mentioned earlier, it also introduced another problem. If the observed doubling in satellite lifespan continues to hold then we must eventually reach a point where lifespan is increasing faster than the passage of time. This would require satellites dying in later years to be launched before satellites dying in earlier years, a seeming contradiction. Eventually we would reach a year for which satellite lifespans ending in that year would be predicted to be longer than the entire history of satellite technology. Since this scenario clearly makes no sense it remains an open problem of how it should be handled. Some progress is explained next.

If we still wish to associate lifespans with end year, when will Moore's law lose its predictive power? For this analysis let Moore's law be defined as:

$$y = a * 2^{\frac{(x-1957)}{b}} \qquad (1)$$

where *a* and *b* are function parameters and *x* represents time and is used to model the current end year. Parameters *a* and *b* are set to 0.549 and 12.17 respectively since this minimizes RMS error (Berleant *et al*. 2019). The input value *x* represents end year and the value *y* is expected lifespan. The first year against which lifespans can be measured is 1957 since that is the year the first satellite was launched, this value is subtracted from *x* and only positive values are considered. For this reason, the historical time span *y* of satellite technology at year *x* is:

$$y = x - 1957 \qquad (2)$$

In order to determine when Moore's law breaks down, we need to determine when the rate that lifespan increases with respect to time equals (immediately following which it will exceed) the rate that time increases with respect to time. In order to do this, we can solve (1) and (2) simultaneously, take the first order derivative and determine the year the two expressions are equal to one another.

$$0.549 * 2^{\frac{(x-1957)}{12.17}} = x - 1957 \qquad (3)$$

Moving both expressions to one side:

$$0.549 * 2^{\frac{(x-1957)}{12.17}} - x + 1957 = 0. \qquad (4)$$

Taking the derivative of the expression:

$$\frac{d}{dx}0.549 * 2^{\frac{(x-1957)}{12.17}} - \frac{d}{dx}x + \frac{d}{dx}1957 = 0 \qquad (5)$$

$$0.549\left[\frac{1}{12.17} * \ln(2) * 2^{\frac{(x-1957)}{12.17}}\right] - 1 = 0. \qquad (6)$$

If we simplify and solve for *x* we obtain:

$$x = 12.17 * log_2\left(\frac{12.17}{0.549 * \ln(2)}\right) + 1957 = 2017.84. \qquad (7)$$

Thus 2017 was the year that lifespans of satellites dying in a given year are predicted to begin increasing faster than the passage of time, a conundrum. What about the point where satellite lifespan is predicted to be greater than the length of the history of satellite technology? If we graph both equations (1) and (2), we can visually observe the points at which they intersect and thus the year that this predicted event might occur. Doing this shows that this point is reached in the year 2046 when average satellite lifespan is predicted to be approximately 89 years, and thus launched prior to 1957, when Sputnik became the first artificial satellite (Figure 3).

So, returning to the earlier point on which law is better for predicting future satellite lifespans based on year of death, Wright's law seems superior simply because (1) Moore's law based on lifespan as a function of end year began failing in principle in 2017 and will reach an even greater level of impossibility in 2046; and (2) launch year cannot work for recent years for which longer-lived craft are still operational.

Discussion

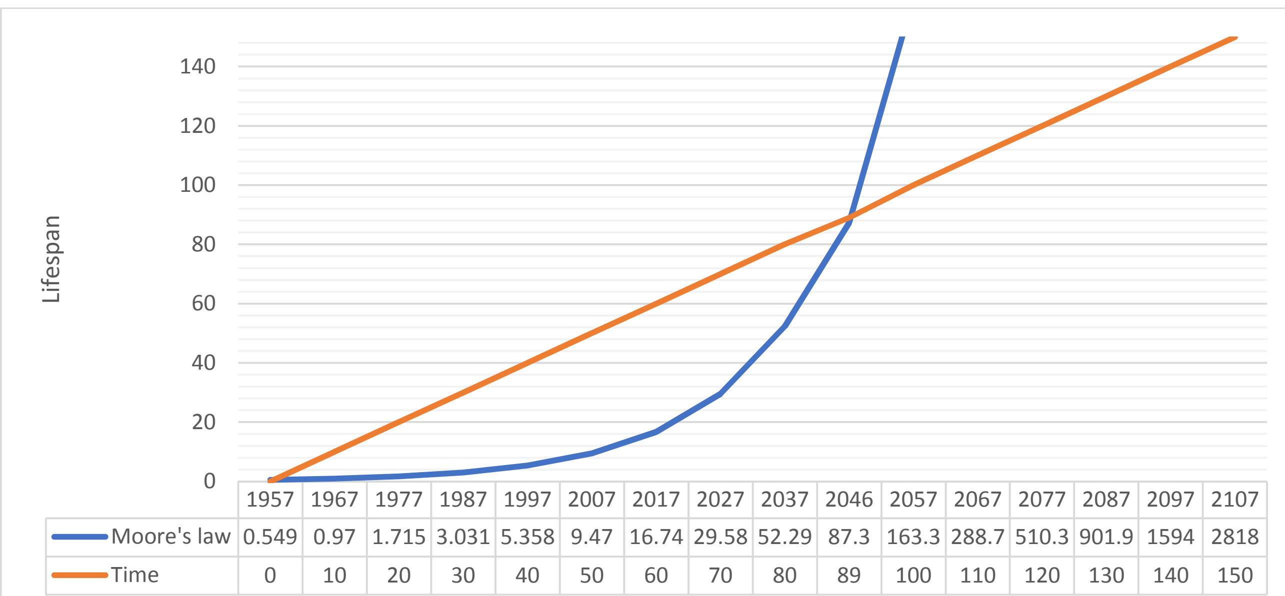
The Moore's law regression was described earlier in equation (1). The simple regression equation for Wright's law is as follows:

$$y = 0.0002446 * ordinality^{1.04} \qquad (8)$$

Where *y* is the average lifespan for satellites ending in that year and *ordinality* is determined by the number of satellites ending in that year and previous years. Our preliminary research suggests that the conundrum associated with Moore's law that was described previously may also apply to the Wright's law regression, although at a much later year, in which case a Wright's law model would not form a principled alternative to a Moore's law model in the case of lifespan as a function of end year. This however remains to be fully investigated.

Conclusions

It may appear that satellite technology has been progressing in an approximately exponential way, perhaps a little less vigorously than a Moore's law



	1957	1967	1977	1987	1997	2007	2017	2027	2037	2046	2057	2067	2077	2087	2097	2107
Moore's law	0.549	0.97	1.715	3.031	5.358	9.47	16.74	29.58	52.29	87.3	163.3	288.7	510.3	901.9	1594	2818
Time	0	10	20	30	40	50	60	70	80	89	100	110	120	130	140	150

Figure 50. Satellite lifespan vs. passage of time, showing a Moore's law crossover

model, but a little more vigorously than a Wright's law model (Figure 1). However, we can confidently predict based on the mathematical deduction presented earlier that the data in coming years must soon break decisively from the Moore's law trend line of Figure 1 and show that lifespan will soon not fit an exponential function of satellite year of death. Importantly however, we have certainly not ruled out the possibility of an exponential trend for some characteristic other than lifespan as a function of year of satellite death.

We plan to empirically verify the analysis we have introduced here against future satellites. Future research is needed to circumvent this mathematical problem and accurately identifying the degree to which space travel is an accelerating technology.

Finally, we close by pointing out that key results presented here should also apply to lifespans of other engineered artifacts besides satellites

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Appendix C: Submission to The Space Review

The following article was published in the space review on December 2, 2019

Trends in technology development in the US and USSR during the Space Race

By Michael Howell, Venkat Kodali, Vladik Kreinovich, Hyacinthe Aboudja, Venkata Jaipal Reddy Batthula, Richard Segall, and Daniel Berleant*

The "space race" ushered in the era of space exploration with an extraordinary government-led rivalry between the United States and the Soviet Union, and their competing political ideologies. This rivalry is rightly regarded as a primary impetus to the early development of space exploration technology. Among the populace, leaving aside politics and even more in the USSR than the US, the inherent appeal of exploring this vast and mysterious "outer space" generated excitement.

The variety of technologies contributing to the space race is immense, but we argue that a useful way to summarize and quantify the data is through the popularly termed Moore's law, or the fitting of an exponential curve to a technology to show its rate of advancement over time. There is a large body of work involving modeling improvement in technological capabilities as exponentially increasing trends, as any search using such queries as "Moore's law" readily reveals. One focus of our research group has been to see if improvements in spacecraft sent on deep space missions can be modeled this way. We have achieved some intriguing results. For example, tuning various parameters of deep space exploration missions to form a composite score for each mission yielded an exponential trajectory for advances in space travel (Hall et al. 2017 [1]). However, a caution in such work is that multi-parameter tuning of the data to maximize goodness of fit to a curve comes with the risk of overfitting. Focusing instead on a single parameter, spacecraft lifespan, reduces this problem and we investigated it in Berleant et al. (2019 [2]). Yet, lifespan as a metric for advancement of a technology leads to intrinsic problems with proper interpretation of relatively recent data (Howell et al. 2019 [3]). In this article, however, we use lifespan on non-recent historical data to gain insight into the space race.

We define spacecraft lifespan as the length of time between launch and the end of a craft's (or all of its major components' including orbiters, rovers, landers, etc.) scientific observations. It turns out that spacecraft lifespan has been showing a marked trend of improvement starting from the earliest days of deep space exploration, and thus appears to be a useful metric for the rate of technology improvement. However, technologies do not advance of their own accord. Scientific, economic, and cultural factors are inherent contributors to the overall picture.

Russian Cultural Precursors

Cultural factors in particular seem to illuminate Russian interest in space exploration and hence the Soviet head start with their famous launch of the first satellite, Sputnik 1. Obviously, economic factors in their space exploration efforts also were important. Despite the brutalities of the Soviet regime, the Soviet Union (USSR) was one of the most rapidly developing economies of the 20th century (Davies 1998 [4]). While this rapid pace of economic growth ended with stagnation in the Brezhnev regime beginning in the 1970s (Service 2009 [5]), the Soviet Union still held the 2nd highest nominal GDP in the world [6]. However cultural factors, while often overlooked, were critical.

There was considerable early interest in space exploration even prior to the formal beginnings of the Soviet space program in a movement that has come to be known as cosmism. Cosmism was a mixture of the occult, religious philosophy, and serious science driven by a utopian vision of what space travel could ultimately mean for humanity. One of the most influential cosmists was Russian philosopher Nikolai Fedorov (or Fyodorov, 1829-1903) whose ideas were published posthumously in *The Philosophy of the Common Task* (Fedorov [7]). A devout Russian Orthodox Christian, he advocated strongly for the use of science to bring about immortality, resurrection of the dead, human enhancement, and colonization of the galaxy (Tandy & Perry [8]). Many of his ideas for achieving these goals bear a striking resemblance in motivation to today's interests in cloning, genetic engineering, and nanotechnology. Fedorov had a profound impact on many Russian intellectuals including Tolstoy and Dostoevsky although he and Tolstoy would eventually become estranged due to religious differences (Zhilyaev et al. [9]; Koutaissoff 1984 [10]).

One of the most important cosmist thinkers was rocket scientist Konstantin Tsiolkovsky. He is famously quoted as saying that Earth is the cradle of humanity, but one cannot live in a cradle forever. Despite little formal education, Tsiolkovsky was responsible for much initial theoretical work in space travel which later laid the foundations for the Soviet space program (Fedorov [7]). In 1897 he carried out early experimental work on aerodynamics in his apartment. In 1929 he developed the concept of the multistage rocket. His other designs included airlocks, space stations, and rocket fuel. His mathematical model of rocket propulsion, derived from earlier work on motion of bodies whose mass varies over time (since using fuel changes vehicle mass), is known as the Tsiolkovsky Rocket Equation (NASA [11]; [12]). Tsiolkovsky had met

Fedorov at the age of 16 and was deeply impressed (Zhilyaev et al. [9]). Like Fedorov, Tsiolkovsky had a philosophical bent and believed his work was laying the foundation for an immortal galactic civilization [13].

Cosmism later found its way into the "Proletkult" movement in Soviet Russia. The goal of Proletkult was to establish a new kind of culture for the Russian working class (Parkinson 2019 [14]). Proletkult was full of aesthetic appeals made to labor, industry, and — space travel (Seifrid 2009 [15]). It remained independent from the Soviet state until 1920 when it was officially adopted by the Ministry of Education. There was a tremendous popular interest in space travel with the Russian media publishing over 250 articles and 30 nonfiction books on the subject between 1921 and 1932 (Parkinson 2019 [14]). 1924 saw the release of the first Russian science fiction film, *Aelita: Queen of Mars*, which depicts a young man traveling to Mars to begin a proletarian revolution (IMDb [16]). The burst of public enthusiasm for space travel tapered off in the 1930s in conjunction with governmental actions that discouraged it including the end of the private publishing industry and promotion of the related but much more practical growing field of aviation, and yet enthusiasts coming of age in the 1920s later became key contributors to the launch of the Soviet space program in the 1950s (Siddiqi 2015 [17]). The new ability to really explore space for the first time found ready support among the people, with songs like "14 Minutes to Start" (e.g. [18]) which achieved greatest popularity in 1962 [19].

As a result of the early success of the Soviet space effort, in particular the launch of the first artificial satellite, Sputnik 1, an interesting phenomenon emerged in the form of competition between the United States and the Soviet Union during a period of rivalry in space exploration termed the "space race." We investigated this from a Moore's law (exponential advancement) standpoint below. Our data points were restricted to deep space missions with an extraterrestrial body as the destination. Thus, Earth satellites are not included in our data analysis. Therefore we begin with the launch of Luna 1 by the Soviet Union in 1959 and end with Phobos 2, again launched by the Soviet Union, in 1989. Certainly many missions were launched after this date but Phobos 2 was the Soviet Union's last deep space flight so at that point the space race was effectively over.

Analysis and results

Lifespans of spacecraft in years were taken from [20] and their logs plotted vs. the date of the end of craft lifespan. With logarithmic scaling of the *y*-axis, an exponential trendline is linear in appearance and thus a linear regression can be fitted for spacecraft lifespan. The x-axis shows date of end of lifespan rather than date of launch because launch date introduces the problem of handling missions that are still in operation, for which the lifespans are therefore not yet known, thus biasing lifespan analyses (Howell et al. 2019 [3]). For example for the space race period, Voyager 1 and 2 are both still operating. Figure 1 shows the resulting regression curves, one for the United States and one for the Soviet Union.

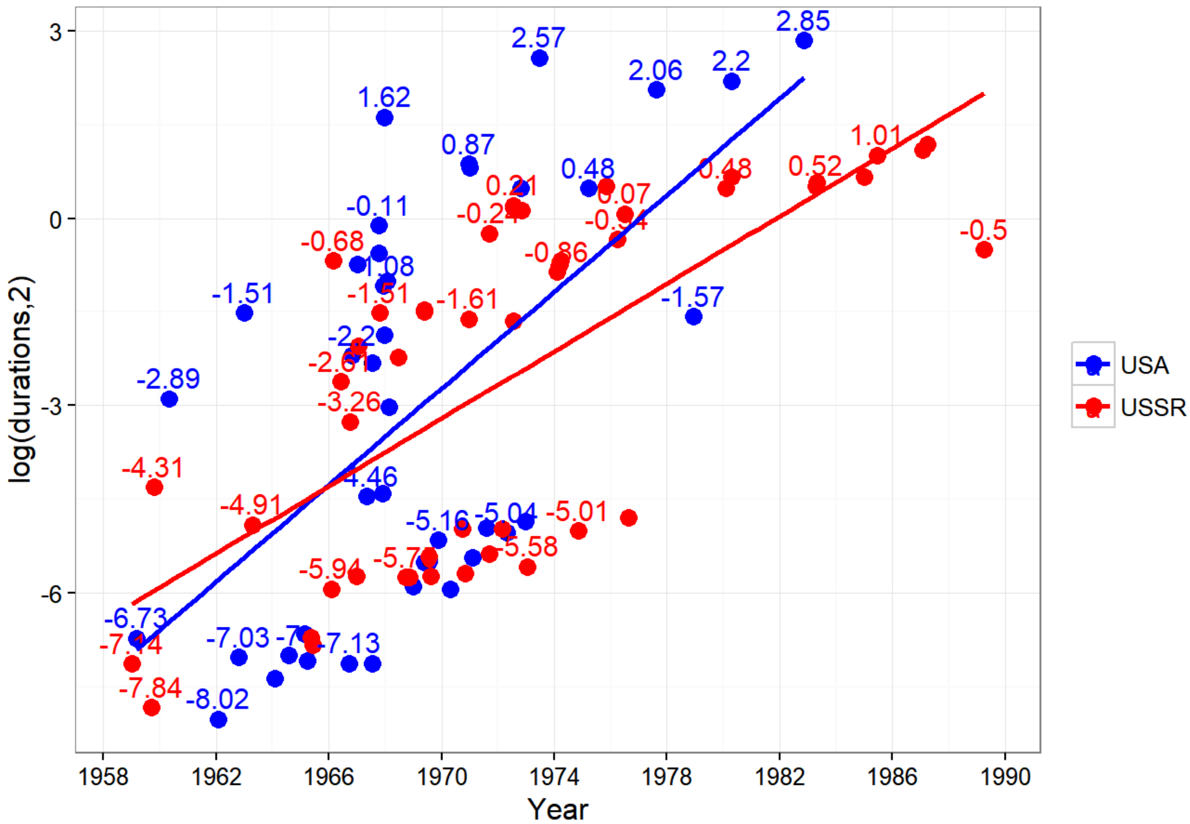

*Figure 51. Plots of log2 of lifespan in years (y-axis) vs. time point of lifespan end (x-axis) for the United States (blue) and the Soviet Union (red). Regression equations are log2(*Lifespan*) = -6.9130 + 0.3873*(Year-1959)* for the US, and *log2*(*Lifespan*) = -6.1794 + 0.2704*(Year-1959) for the USSR.*

We can see from the lower left portion of Figure 1 that the USSR possessed an early lead over the US in spacecraft lifespans. Despite this head start, however, the United States experienced a faster rate of progress with spacecraft lifespans increasing by an average 30.8% each year [21], for a doubling time of 2.6 years [22], compared to the USSR which showed a lower 20.6% yearly increase (a doubling time of 3.7 years).

Viewing changes in lifespan over time as a measure of spacecraft technological progress, this quantitatively illustrates the higher rate of improvement based on which the US is generally viewed as having in some sense “won” the space race. Viewing space exploration as a grand activity of the human race, the small steps and giant leaps of both nations back then, and a growing number of nations today, were sparked first by Russian cosmism and then by the space race rivalry which helped form the technological foundations of the space exploration of today and tomorrow.

Acknowledgments

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[21] Without loss of generality one may choose a time point t such that *lifespan*(t)=$2^{0.3873t}$. Then 1 year later,
lifespan(t + 1)=$2^{0.3873(t+1)}$
= $2^{0.3873t + 0.3873}$
= $2^{0.3873t} * 2^{0.3873}$
= *lifespan*(t) * $2^{0.3873}$
= *lifespan*(t) * 1.308
= *lifespan*(t) + 30.8%

[22] To determine the doubling time for the US, the slope was delta y/delta x = 0.3873. Then 1/slope = delta x/delta y = 1/0.3873 = 2.58/1. Each unit on the y axis represents a doubling because it is scaled logarithmically with base 2, so 2.58 years on the x axis is the doubling time.