

# UNCERTAINTY QUANTIFICATION IN A MODEL ELECTRIC ENERGY BIDDING PROBLEM: A MULTI-CRITERIA DECISION MAKING APPROACH

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**Abstract** – This paper explores a wide range of different decision criteria for the bidding problem when severe uncertainty about inputs requires expressing them as bounded families of distributions. A GENCO maximizes its expected returns given its own risk profile and its perception of market risks. This can be formulated as a multi-criteria optimization problem. The set of different decision criteria examined illustrates the variety of different approaches possible, the different conclusions these approaches produce, and the bidder's need to understand goals and information availability in order to rationally decide on a bid to best meet the goals of the organization. This paper highlights the different possible decision criteria and uses Analytic Hierarchy Process to determine the highest priority bid based on qualitative and quantitative analyses.

**Keywords:** *Bidding Strategies, Information Gap Decision Theory, Decision Support Systems, Optimization, Multi-Criteria Decision Making, Analytic Hierarchy Process.*

## 1 INTRODUCTION

The net gain for a GENCO is dependent on the volatility of the Market Clearing Price (MCP) which in turn is determined by the strategies employed by other market participants and by uncertain market conditions. Therefore, deciding what to bid in an electricity auction typifies the large class of problems requiring decisions despite the possibility of severe uncertainty about important problem parameters.

New ideas for innovative solutions to such problems are arising from recent activity in uncertainty quantification (also called 2<sup>nd</sup>-order uncertainty, epistemic uncertainty, and other terms). Traditionally, models proposed have represented uncertainty about continuous variables using probability distributions. This enables well-known techniques like Monte Carlo simulation. Other approaches to uncertainty have also been applied to electric energy industry and other problems, most notably fuzzy methods and intervals. Methods consistent with the classical, mathematically well-founded framework of probability, however, have stood the test of time and are undoubtedly here to stay. Because knowledge that supports bidding decisions is often limited by what information is available, inference under conditions of epistemic uncertainty is important to do in order to support optimized bidding. In this paper,

such uncertainty is quantified as bounded families of probability distributions.

Applying bounded families of distributions to electric power bidding problems exemplifies what may be expected in engineering from applying uncertainty quantification. The growth in interest in uncertainty quantification engendered by such expectations is illustrated, for example, by recent journal special issues [1-2]. Applications to problems in electric power [3-4] follow naturally from the insights and investigations of other researchers, who have found that uncertainty quantification has applicability to power problems characterized by severe uncertainty. Prominent techniques include intervals [5-6] and fuzzy methods [7-8]. The well-recognized need for decisions in the presence of severe uncertainty, coupled with the grounding of our approach in the mathematically well-founded theory of probability, support its use in addressing important problems in electric power. Guidance can then be obtained regarding bidding decisions under conditions of uncertainty in which standard methods would require extra, unjustified assumptions.

While one can obtain different bidding decisions based on different decision criteria, to obtain a single optimal bid decision is highly dependent on qualitative and quantitative analyses. This paper addresses this problem by using the Analytic Hierarchy Process (AHP) to guide decision maker in establishing the most significant bid decision.

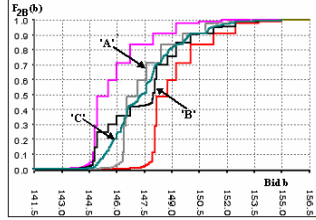
## 2 MODELING THE PROBLEM

We wish to optimize bidding in the presence of uncertainty about the applicable distribution. Figure 1 shows an example. In a figure of this type, the left- and rightmost cumulative distribution functions (CDFs) could arise for at least three reasons: (1) as confidence limits [9]; (2) as bounds on the distribution of a function of two (or more) random variables whose dependency relationship(s) are unspecified, e.g. using the DEnv algorithm [10]; or (3) as bounds on the set of different distributions estimated by different experts. The stepped appearance in the figure could be due to the graininess of data in the Kolmogoroff case, or to discretization in the computations in the case of the DEnv algorithm. In the case of composition of expert estimates the curves would probably be smooth instead of stepped. However

whether it is stepped or smooth does not impact the subsequent discussion.

Consider an example incorporating Figure 1. We take the perspective of GENCO 1, which is bidding against GENCO 2 to sell electricity with the following problem parameters.

- Demand  $X_D=1000\text{MWh}$
- Cost to GENCO 1=\$40/MWh
- GENCO 2 has two generators,  $G_{2A}$  and  $G_{2B}$
- Capacity of  $G_{2A}$  is  $X_{2A}=300\text{MWh}$
- Capacity of  $G_{2B}$  is  $X_{2B}>700\text{MWh}$



**Figure 1:** Bounds on the cumulative distribution for the bid that will be made by GENCO 2, for power from its generator  $G_{2B}$ . Curve A is the horizontal average of the left- and rightmost curves, curve B is the vertical average, and curve C is the fixed-point average.

Careful inspection shows that underbidding  $G_{2A}$  with an offer of 1000MWh will result in selling the entire 1000MWh, with GENCO 2 unable to sell from either  $G_{2A}$  or  $G_{2B}$ . However, this will have a lower expected monetary value (EMV) to GENCO 1 than attempting to underbid  $G_{2A}$  with a block of 300MWh and  $G_{2B}$  with a block of 700MWh. A still better strategy is to underbid  $G_{2B}$  and sell 700MWh at a relatively high price while permitting GENCO 2 to sell 300MWh at the relatively low price [4]. However, this strategy is consistent with a range of different bid prices. Given this strategy, we wish to determine what price GENCO 1 should bid to its best advantage.

### 2.1 Implications for Bidding

If GENCO 1 bids below the left tail of the left bounding curve in Figure 1, it underbids GENCO 2's  $G_{2B}$ , therefore selling 700 MWh with probability 1. For bids between the left tail of the leftmost curve and the right (upper) tail of the rightmost curve, GENCO 1's EMV for a bid is:

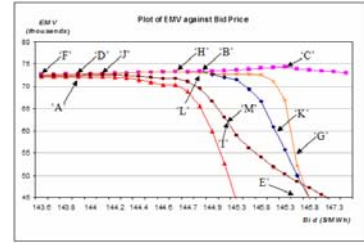
$$EMV = p_{win}(b) * 700 * (b - 40)$$

where  $b$  is the bid amount and 40 is the per-MWh production cost given above. The probability of winning the auction is  $p_{win}(b) = 1 - F_{2B}(b)$ , where  $F_{2B}(\cdot)$  is the cumulative distribution for the bid predicted from competitor GENCO 2 for power from  $G_{2B}$ . The form of this distribution is uncertain, however it must fall between the left and right curves of Figure 1. Each possible curve for  $F_{2B}(b)$  implies a different curve for  $EMV(b)$ . Some of these are shown in Figure 2.

If the correct EMV curve is known, then the horizontal axis coordinate of its highest point defines the optimum bid. Figure 2 shows that the optimum bid differs for different EMV curves. Example optimum bids

shown in Figure 2 are the horizontal axis coordinates of triangular point A, x-shaped point B, and point C. Thus, although the best bid may be easily found for a particular EMV curve, the situation is less clear when the proper EMV curve is unknown. Some other points in Figure 2 also deserve mention. Point E (below the visible portion of the graph) is the point on the low curve with the same horizontal axis value as point C on the high curve. Point F is where the curves start to diverge. And finally, if the top and bottom EMV curves are vertically averaged, curve M is obtained.

In order to address bidding when the right EMV curve to use is unknown, we start by observing that the family of possible EMV curves (i.e. those that are implied by cumulative distributions within the bounds shown in Figure 2) is bounded from below by one such curve. We term this the pessimistic curve. Similarly the family is bounded from above by another EMV curve termed the optimistic curve.



**Figure 2:** Some possible curves for the EMV as a function of bid value. Each EMV curve corresponds to some distribution that predicts the competing bid of GENCO 2 for its generator  $G_{2B}$  (see Fig. 1).

Suppose the best bid is chosen under the assumption that the pessimistic curve applies. The maximum point of that curve, whose coordinates state the best bid and its EMV, is triangular point A in Figure 2. If instead the optimistic curve applies, then the EMV of the same bid is higher (square point D). While this higher EMV is welcome, it is not as high as it would have been had we only known the optimistic curve applied. Then we could have chosen a different bid with an even higher EMV (point C). Hence the suboptimality of the bid is potentially as great as the difference in the heights of points D and C.

## 3 DECIDING ON A BID UNDER SEVERE UNCERTAINTY

When such bounded EMV curves arise, there are many different ways to generate the bidding decision such as the following ten decision criteria (DC1-DC10). They can be categorized by decision criteria based on extreme scenarios (DC1-DC3), decision criteria based on averaging scenarios (DC4-DC6), decision criteria based on risk (DC7-DC10). Each criterion has its own conditions of applicability – and its own answer.

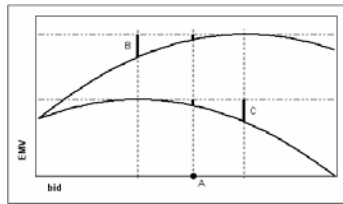
### 3.1 DC1: Minimize Potential Suboptimality

The principle here is to minimize the maximum possible suboptimality. By suboptimality, it is meant the

worst case amount by which the EMV of a bid might fall short of what it would be if the actual EMV curve were known. Figure 3 illustrates the bid resulting from such an analysis. In the figure, the length of a heavy vertical line segment indicates the worst case suboptimality of the corresponding bid, which is indicated by the placement of a vertical dotted line. Optimizing the EMV of the low EMV curve results in a bid whose worst case suboptimality is the length of line segment B. Bidding to optimize the EMV of the high EMV curve results in line segment C. Both B and C are longer than either of the two heavy line segments associated with bid A. For an EMV curve family like this, a graphical way to apply this decision criterion is to find the bid for which the suboptimality, if the low EMV curve applies, equals the suboptimality given that the high EMV curve applies. This decision criterion may be particularly appropriate in high-stakes scenarios in which it is desired to minimize risk.

### 3.2 DC2: Maximize the Minimum Possible EMV

If the EMV of a bid is estimated from the lowest (most pessimistic) curve in the family of plausible EMV curves, then the estimated best bid and its EMV is the maximum of that curve. In Figure 2 that is the bid of \$143.92/MWh (point A). The actual EMV of that bid, which may be governed by a higher EMV curve, will then be at least as high. This is conservative in the sense that the actual EMV of the bid is at least as high. This strategy however is problematic for two reasons. First, the actual EMV curve is probably one of the infinitely numerous ones that are higher than the lowest one, and they have maxima at bids above \$143.92/MWh. Therefore, a bid of \$143.92+ $\epsilon$ /MWh probably will have a higher EMV than \$143.92/MWh. Second, even a very risk averse bidder for whom winning has overriding importance would not bid \$143.92/MWh. Such a player would instead place a bid just below the bottom tail of the leftmost curve in Figure 1. This is approximately \$143.2/MWh.



**Figure 3:** A family of EMV curves (lowest and highest shown), and three bids represented by vertical dotted lines. The length of the longest solid line segment associated with a bid is that bid's worst case suboptimality.

### 3.3 DC3: Maximize the Maximum Possible EMV

This approach is the conceptual opposite of the one just described. Here the bidder is guided by the optimistic EMV curve instead of the pessimistic curve. In this strategy, the optimal bid is \$146.30/MWh, the horizontal axis coordinate of point C in Figure 3. However, the consequences if another EMV curve much below the

top one applies could be serious. Many of the EMV curves in the family are dropping off precipitously at that bid value, leading to a serious risk of a very low EMV if one of the lower EMV curves is the correct one, such as the EMV of point E. Even a very risk-seeking bidder might not use this strategy because yet higher bids could still be successful (as long as they are below \$155/MWh, the end of the upper tail of the rightmost curve in Figure 1).

### 3.4 DC4: Use Horizontal Averaging

This method computes a curve estimating the CDF of the competitor's bid by connecting points that are calculated as follows. Any given value of  $F_{2B}$  in Figure 1 intersects the left- and rightmost curves at corresponding bid values, call them  $b_l$  and  $b_r$ . Specify a new value  $b = (b_l + b_r)/2$  as a coordinate of a new point  $(b, F_{2B})$ . Done enough times, the result is curve A in Figure 1. This curve corresponds to EMV curve G in Figure 3, for which the best bid and its EMV are shown as point H.

### 3.5 DC5: Use Vertical Averaging

The horizontal averaging strategy just described has a vertical dual: for any given value of  $b$  on the horizontal axis, the vertical axis values of the left- and rightmost curves are both identified. Their mean, and  $b$ , form the coordinates of a point on the vertically averaged curve. The result is curve B in Figure 1. This curve implies curve I in Figure 2, with a maximum at point J that defines its best bid and corresponding EMV.

### 3.6 DC6: Use Fixed-Point Averaging

The CDF curves predicting the competitor's bid implied by horizontal vs. vertical averaging of the left- and rightmost curves are different (Figure 1, curves A and B). These average curves have complementary desirable and undesirable features. The horizontally averaged curve has an S-shape but the vertically averaged curve has a wavy form that seems counter-intuitive (though, in principle, possible). On the other hand, the vertically averaged curve has tails that reflect the plausibility of competitor bids that are extreme but within the bounding curves, while the horizontally averaged curve does not. Thus a third form of averaging was designed that has the advantages of both.

This averaging method takes the horizontal and vertical average curves as inputs, and averages those curves. However, if this new averaging step was done by vertical averaging, the result would be different than if it was done by horizontal averaging. Therefore, the average curves (A and B in Figure 1) are averaged both ways to obtain two new curves, these new curves are then again averaged both ways, and the process iterated until the two curves resulting from each iteration converge. The result (curve C in Figure 1) is both S-shaped and has the desired tail property. This is termed the fixed-point average curve, and the procedure, fixed-point averaging.

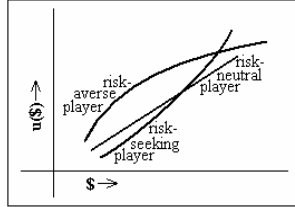
The fixed-point average curve implies a corresponding EMV curve (labeled K in Figure 2). This curve has

a maximum point (labeled L in Figure 2) which gives the maximum EMV and its corresponding bid amount, as estimated through fixed-point averaging.

### 3.7 DC7: Use Bid Utility Instead of Bid EMV

The goal in choosing a bid price is ultimately to optimize its utility to the business. For bids to sell blocks of power this might mean maximizing the expected monetary value (EMV) of a bid. However, a full analysis would need to recognize that the goal of maximizing EMV may be tempered by risk position. For example, individuals who make bids may wish to avoid jeopardizing their bidding records with risky bids. As another example, risk aversion may affect higher level decisions of sufficiently high value, such as those relevant to value at risk (VaR), profit at risk (PaR), capital budgeting, etc. To account for risk, the utility of bids rather than their EMV should be maximized. This may require transforming EMV curves (Figure 2) into utility curves based on risk profile. Then the solution approaches described earlier, which refers to EMV curves, can often instead be applied to the corresponding utility curves.

EMV curves may be transformed into corresponding utility curves by transforming the points on those curves (the EMVs of bids) into utilities of bids. The first step in converting an EMV to a utility is specifying the risk profile of the player, GENCO 1 in this case, as a utility function. Figure 4 shows representative examples of utility functions.



**Figure 4:** Three risk profiles. Risk neutrality gives a constant slope utility function, risk aversion a concave-down one, and risk attraction a concave-up one.

A utility function,  $u(x)$ , describes utility as a function of monetary quantity. It expresses the player's subjective perception of the value of objective monetary amounts. A utility function can model the fact that, for example, a risk averse player prefers getting \$10,000 to a 50% chance of getting \$20,000, even though both options have the same expected monetary value (EMV). Thus the utility of \$10,000 to this player exceeds 50% of the utility of \$20,000 + 50% of the utility of \$0. In general, the utility of a gamble is the average of the utilities of the possible outcomes weighted by their probabilities. Thus for this player,  $u(\$10,000) > (\frac{1}{2})u(\$20,000)$ . This implies a utility function of decreasing slope and a risk averse player.

For the example bidding problem, a given CDF in the family of CDFs (Figure 1) predicting competitor GENCO 2's bid associates each bid amount  $b$  with a cumulative probability  $p = F_{2B}(b)$ . Thus if GENCO 1

bids at  $b$  the probability is  $p$  that GENCO 2's bid is lower. Therefore GENCO 1 will win the auction with probability  $1-p$ . If GENCO 1 wins, the resulting sale will have a certain monetary value  $v_b$ . If GENCO 1 loses, the model assigns that result a value of 0. The utility of the bid,  $u_b$ , is thus  $u_b = (1-p)u(v_b) + p u(0)$ . Its monetary equivalent is then  $u^{-1}(u_b)$ . If risk is an insignificant consideration then risk-neutrality applies, so  $u(v) = v$ . Then the utility of a bid equals its EMV, that is,  $u_b = (1-p)u(v_b) + p u(0) = (1-p)v_b + p \cdot 0 = \text{EMV}_b$ . Taking risk into consideration then has no effect on the decision of what to bid. Risk neutrality might apply if the amount of money involved is small from the bidder's standpoint, such as one auction out of many occurring over time. On the other hand, risk would be a consideration for a large enough bid, for example, if it was for a significant capital investment like building a windmill farm.

### 3.8 DC8: Use EMV Utility Instead of Bid Utility

Given a bid amount  $b$  and the associated  $\text{EMV}(b)$ , the monetary value of  $n$  such bids will be close enough to  $n \cdot \text{EMV}(b)$  to within an acceptable probability for sufficient  $n$ . Thus  $n \cdot \text{EMV}(b)$  approaches not an EMV but an actual monetary value, as a statistical property of a large enough number of bids. The risk in this scenario is not losing a given auction. Rather, the risk is of choosing bids with relatively low EMVs due to basing them on the wrong CDF in the CDF family.

Given plans for  $n$  auctions before a re-examination of bidding policy, a single-bid EMV of  $m$  has a utility of  $u(n \cdot m)/n$  irrespective of the probability of winning. Since any EMV can thus be converted into a utility value, any EMV curve can be transformed into a utility curve. This enables transforming a family of EMV curves such as those in Figure 2 into utility curves. These utility curves might normally be expected to have some resemblance to their original EMV curves. For example, the maximum of each utility curve will be at the same bid value as the maximum of the EMV curve of which it is a transformation. Yet the utility curves will not be identical to the EMV curves (unless risk neutrality applies). Therefore various decision criteria described earlier, if given utility curves as inputs, will likely produce recommendations for bids that differ to some degree from the recommendations they make when applied to the corresponding EMV curves.

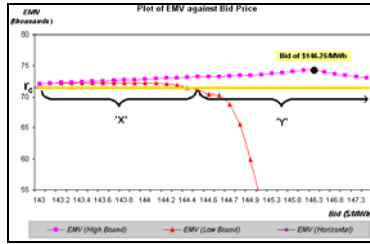
### 3.9 DC9: Use Information Gap Theory

Information Gap Theory [11] is useful for making decisions in cases where uncertainty is described with bounds, but the probabilistic structure within those bounds is not specified such as Figure 1. Rather than attempting a form of optimization as was done in earlier decision criteria, the appropriate goal in using information gap analysis here is to ensure that the EMV of a bid meets or exceeds a given minimum value. An information gap model for this results in a *robustness function* that helps identify bids meeting that requirement. An information gap model can also identify the additional

information that would be needed to reduce the uncertainty enough to ensure that other, more desirable bids, meet that requirement.

For example, various bid amounts in Figure 2 have high EMVs for EMV curves near the highest EMV curve, making them potentially desirable bids. However some of these bids also lead to unacceptably low EMVs for EMV curves near the lowest EMV curve, making them too risky. If additional information was obtained that ruled out such low EMV curves, some overly risky bids would then become feasible. The cost of obtaining such information could be weighed against its potential benefit and a decision then made about whether to obtain it. However such information, once obtained, might rule out high EMV curves instead, thereby also ruling out bids that it was hoped would become feasible. Results of an information gap analysis for the bidding problem are shown in Figure 5.

More formally, an information gap model requires defining a number of items: decision variable (bid  $b$ ), uncertain variable (CDF), nominal value of uncertain variable ( $\tilde{w}=0.5$ ), uncertainty parameter ( $\alpha$ ), uncertainty model ( $\mathbf{u}(\alpha, \tilde{w})$ ), reward function (EMV), critical reward ( $r_c$ ), and robustness function ( $\hat{\alpha}(b, r_c)$ ). For more detailed explanation on each item, please refer to [12].



**Figure 5:** Bounding EMV curves (from Figure 2), a given minimum acceptable reward level  $r_c$ , and two ranges of bids, X and Y. Bids in range X guarantee EMVs of at least  $r_c$ , while bids in range Y might have EMVs of at least  $r_c$ , but also might not.

From Figure 5 some facts may be deduced about  $\hat{\alpha}(b, r_c)$  for a range of bid amounts, given a minimally acceptable reward  $r_c$ . For bid amounts in region 'X' the EMV is above  $r_c$  for all EMV curves corresponding to CDF  $L$ ,  $R$ , or any average of  $L$  &  $R$ , weighted or not. Therefore weight  $w$  could be from 0 to 1, meaning that  $\hat{\alpha}(b, r_c)$ , the maximum allowable deviation  $\alpha$  from nominal weight  $\tilde{w}$ , is  $\tilde{w}$  in the downward direction (in which case  $0 \leq w \leq \tilde{w}$ ) and  $1 - \tilde{w}$  in the upward direction (in which case  $\tilde{w} \leq w \leq 1$ ). Thus,  $r_c$  will be safely met for any bid in region 'X.' However it may be desired to consider bidding higher (in range 'Y') in order to attempt to reap the benefits of possibly greater EMVs. Bids in range 'Y' in Figure 5 would be guaranteed to have an EMV of at least  $r_c$  if new information is obtained which rules out values of  $w$  that are too close to 1 (thereby moving the worst-case EMV curve upward).

Thus, new information about  $w$  may be sought that would permit bids above region 'X.' This information, once obtained, might or might not do this, depending on what values of  $w$  the new information rules out.

To summarize, the result of an information gap analysis is a description of what new information (if any) would be required, for any specified bid amount, to guarantee at least a given EMV. Thus guidance is provided about both acquiring new information and making bids.

### 3.10 DC10: Apply VaR/PaR With Bernoulli Processes

Bernoulli processes may be used to constrain acceptable bid amounts based on Value at Risk (VaR) or Profit at Risk (PaR). Given a maximum loss (VaR) or minimum profit (PaR)  $v$ , a corresponding certainty factor  $p$  representing the minimum tolerable probability of meeting requirement  $v$  by a time  $t$ , and a function  $F$  relating a bid amount to the probability of winning the auction, an acceptable range for bids may be deduced. The outcome of a particular auction will change the parameters of the problem because there are fewer bids remaining until time  $t$ ; also, winning an auction changes the amount of money that may be risked over the course of the remaining bids. Consequently, the range of acceptable bid amounts will tend to vary from one auction to the next.

Under severe uncertainty, exact probabilities of exceeding specific values of loss (Value at Risk, or VaR), or failing to meet specific values of profit (Profit at Risk, or PaR) can be impossible to compute. Applying the Bernoulli process approach in the presence of 2<sup>nd</sup>-order uncertainty about CDFs requires accounting for that uncertainty in deciding on bid values (Figures 1 & 2). While uncertainty in the EMV of a given bid has been addressed using various decision criteria described earlier, the 2<sup>nd</sup>-order uncertainty in the problem description can alternatively play out as uncertainty in the probability of winning an auction for a given bid. Viewing each auction as a Bernoulli trial, this uncertainty about the probability of winning leads to a range of possible probabilities of meeting VaR or PaR requirement  $v$  in the remaining Bernoulli trials.

If every value in the range of probabilities is below the desired certainty factor  $p$ , then the bid is allowable. If no bid is allowable, acquiring more information might reveal a range of acceptable bids. If not, still more information might be sought. If it is possible to show that additional information could not lead to any acceptable bids, then the conclusion is that the VaR or PaR objective cannot be achieved with the desired certainty factor  $p$ , necessitating some form of damage control. If analysis does provide a range of acceptable bids for the next auction, acquiring more information might expand the range. The details of what information is required may be determined by an information gap analysis (one example of which was given earlier). Ultimately, a bid within the acceptable range must be chosen. This may be done using one of the other applicable optimization methods described earlier.



### 3.11 Decision Criteria Summary

With ten decision criteria that include maximizing worst-case EMVs, maximizing expected EMVs, and converting EMVs to utilities using risk profiles, different results can be obtained and each can be better or worst than the other in its own perspective. Even though ten decision criteria have been highlighted, the decision maker may only consider few that are most significant. For example, suppose GENCO 1 decides to utilize DC1-DC6. The resulting bids and expected profit can be summarized in Figure 6. While these bids may only differ in the order of \$0.10/MWh, the expected profit may differ in a large range that impacts the profitability of GENCO 1.

From the six decision criteria, one may decide to bid at \$146.30/MWh to gain the maximum expected profit of 74315.50, however this correspond to the high bound of the EMV curves in Figure 2. Unless GENCO 1 is risk-seeking, it would not choose this as the optimal bid because the worst case scenario may cause a major loss. Similar explanations can be applied to other five different decision criteria based on the discussion above. Therefore, a multi-criteria decision making approach is needed and will be discussed in the following section.

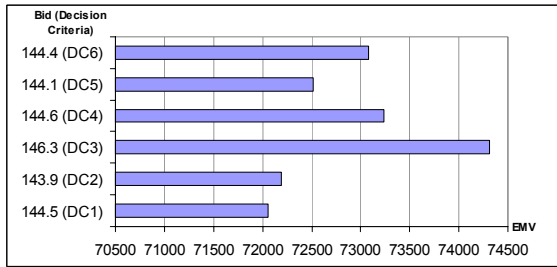


Figure 6: EMV values for DC1-DC6

## 4 USING ANALYTIC HIERARCHY PROCESS TO MAKE THE BEST BID DECISION

Developed by Dr. Thomas Saaty in 1970 [13], the Analytic Hierarchy Process (AHP) is a powerful and flexible decision making process to help the decision maker set priorities and make the best decision when both qualitative and quantitative aspects of a decision need to be considered. AHP provides a clear optimal rationale by reducing complex decisions to a series of one-on-one comparisons.

Suppose that a value function,  $v(y)$  is the weighted sum of the corresponding outcome  $y_i$ , and the weight ratio is defined as:

$$w_{ij} = \frac{w_i}{w_j} \quad \forall i, j$$

where  $w_{ij} = w_{ji}^{-1} = w_{ij} w_{kj}$  and it is consistent. If these two conditions hold, the matrix of weight ratios can thus be described as:

$$W_{q \times q} = \begin{pmatrix} \frac{w_1}{w_1} & \frac{w_1}{w_2} & \dots & \frac{w_1}{w_q} \\ \frac{w_2}{w_1} & \frac{w_2}{w_2} & \dots & \frac{w_2}{w_q} \\ \dots & \dots & \dots & \dots \\ \frac{w_q}{w_1} & \frac{w_q}{w_2} & \dots & \frac{w_q}{w_q} \end{pmatrix}$$

Notice that the rank of  $W$  is one since each row of  $W$  is a multiple of the first row and there is only one non-zero eigenvalue  $q$ . This is due to the fact that  $w_{ii} = 1$  and the sum of all eigenvalues is equal to the

trace of  $W$ ,  $\sum_{i=1}^q w_{ii} = q$ . Since  $Ww = qw$ ,  $w$  must be the eigenvector of  $W$  corresponding to the maximum eigenvalue  $q$ .

To overcome fixed weight problem, Saaty proposes to estimate the weight ratio by a constant  $a_{ij}$ . Let  $A = [a_{ij}]_{q \times q}$  be the matrix of components  $\{a_{ij}\}$ . Since  $w_{ij} > 0$ , we shall assume  $a_{ij} > 0$ . Since  $w_{ij} = w_{ji}^{-1}$ , then the only elements  $a_{ij}$ , where  $j > i$  need to be considered. Since  $A$  is found as an approximate for  $W$ , when the consistency conditions are almost satisfied for  $A$ , one would expect that the normalized eigenvector corresponding to the maximum eigenvector of  $A$ , denoted by  $\lambda_{max}$ , will also be closed to  $w$ . There are two theorems associated with these assumptions and conditions (when the matrix is consistent):

**Theorem 1:** The maximum eigenvalue  $\lambda_{max}$ , of  $A$  is a positive real number. Let  $\hat{w}$  be the normalized eigenvector corresponding to  $\lambda_{max}$  of  $A$ . Then  $\hat{w} > 0$  for all  $1 \leq i \leq q$ .

**Theorem 2.** The maximum eigenvalue of  $A$  satisfies the inequality  $\lambda_{max} \geq q$ .

Assuming that we have  $q$  objectives and we want to construct a scale, rating these objectives as to their importance with respect to the decision, as seen by the analyst. The decision maker has to compare the objectives in paired comparisons. If objective  $i$  is compared against objective  $j$ , the values of  $a_{ij}$  and  $a_{ji}$  are assigned as follows:

- $a_{ij} = a_{ji}^{-1}$
- If the objective  $i$  is more important than objective  $j$  then  $a_{ij}$  gets assigned a number as follows:

Intensity of relative importance	Verbal definition
1	Equal importance
2	Equal to moderate importance
3	Moderate importance
4	Moderate to strong importance
5	Strong importance
6	Strong to very strong importance

7	Strong importance
8	Strong to extreme importance
9	Extreme importance

**Table 1:** Scale of relative importance.

Assuming that we are deciding to bid high (DC4), medium (DC6) or bid low (DC5) based on the results obtained from the averaging methods (Table 2).

Averaging methods	Optimal Bid (\$/MWh)	EMV (\$)	Maximum possible loss (\$)
DC4: High (Horizontal)	144.63	73238	2913.60
DC6: Medium (Fixed-point)	144.40	73080	1632.7
DC5: Low (Vertical)	144.10	72512	358.71

**Table 2:** Comparison of averaging methods.

Suppose the decision factors to be considered are to maximize expected profit (EMV) and to minimize loss (LOSS), the priorities need to be assigned based on these two factors. The results are illustrated in Table 3. First, the intensity of relative importance needs to be assigned (Table 1) before normalizing them to calculate the relative priorities. We will illustrate on calculating the priority for bidding high. The normalized values are:

$$[\text{High, High}] = \frac{1}{1+1/5+1/9} = 0.7627$$

$$[\text{High, Medium}] = \frac{1}{5+1+1/7} = 0.8139$$

$$[\text{High, Low}] = \frac{1}{9+7+1} = 0.5294$$

$$[\text{High, Priority}] = \frac{0.7627 + 0.8139 + 0.5294}{3} = 0.7020$$

EMV	High	Medium	Low	Prior-ity
High	1	5	9	0.7020
Medium	1/5	1	7	0.2424
Low	1/9	1/7	1	0.0556
LOSS	High	Medium	Low	Prior-ity
High	1	1/6	1/8	0.0633
Medium	6	1	1/4	0.2584
Low	8	4	1	0.6782

**Table 3:** The relative priorities for EMV and LOSS

With that, another iteration of relative priorities calculation will be performed on EMV and LOSS to determine the single decision with highest priority. This is shown in Table 4.

Relative priority	EMV	LOSS	Importance
High	0.7020	0.0633	0.5743

Medium	0.2424	0.2584	0.2456
Low	0.0556	0.6782	0.1801

**Table 4:** Final importance if EMV is 4 times preferred over LOSS according to the scale in table 1.

Results also show that if EMV is preferred over LOSS for any scale in Table 1, bid high is preferred over bidding medium/low. If LOSS is more important than EMV, bid Low is preferred over the others. Medium is always not preferred in this method, which is intuitive because EMV and LOSS prefers the extremes than the median or averaged of bids. Other decision criteria can be added to determine the bid with the highest priority with similar calculations.

## 5 CONCLUSION

Quantification of uncertainty in electric energy bidding problem is significant not only due to the volatility of the decision variables but multiple decision criteria can lead to multiple answers. As discussed in this paper, there are many ways for a decision maker to decide on the optimal bid depending on which decision criteria they employ. When the decision criteria can be as straightforward as the extreme scenarios and averaging methods (DC1-DC6), one can just bid based on the results obtained from the specific criteria one chooses. However, this may not exhaust the total revenue that could be obtained simply because there is a better solution if the actual curve is biased towards the high or low EMV curves.

One way to justify how well the decision maker can perform is to quantify the uncertainty. This can be accomplished by applying different probabilistic methods or risk analysis methodologies based on decision criteria. When these different decision criteria give different answers, a decision may still be necessary. One way to identify one is to examine the premises of the criteria and rule out those whose premises are not consistent with the goals and perspectives of the decision-maker. Utility-based criteria might be eliminated from consideration in favor of EMV-based criteria, or vice versa. In general however, more than one criterion recommending different decisions may stubbornly remain plausible. This is in the nature of severe uncertainty. Yet a decision may nevertheless be required. Multi-criteria decision making approach such as AHP can be used to rank order the importance of each decision over the other and determine the decision that gives the highest priority.

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